Non-perturbative features of effective field theories

Luigi Carvalho Ferreira

Instituto de Física

Universidade Federal Fluminense

Supervisor

Dr. Antônio Duarte Pereira Junior

Co-supervisor

Dr. Rodrigo Ferreira Sobreiro

In partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Physics

Ficha catalográfica automática - SDC/BIF
Gerada com informações fornecidas pelo autor

F383n Ferreira, Luigi Carvalho
Non-perturbative features of effective field theories / Luigi Carvalho Ferreira. - 2023.
165 f.: il.

Orientador: Antônio Duarte Pereira Junior.
Coorientador: Rodrigo Ferreira Sobreiro.
Tese (doutorado)-Universidade Federal Fluminense, Instituto de Física, Niterói, 2023.

1. Problema de Gribov. 2. Teoria de Yang-Mills. 3. Modelos efetivos não-Abelianos. 4. Confinamento. 5. Produção intelectual. I. Pereira Junior, Antônio Duarte, orientador. II. Sobreiro, Rodrigo Ferreira, coorientador. III. Universidade Federal Fluminense. Instituto de Física. IV. Título.

CDD - XXX

Bibliotecário responsável: Debora do Nascimento - CRB7/6368

Acknowledgements

I would like to thank first of all to my advisor Antônio Duarte Pereira Junior and to my co-advisor Rodrigo Ferreira Sobreiro for having guided me during the last four years. Thanks specially for the patience and for the corrections that helped me a lot in the elaboration of this thesis. I thank to the colleagues and friends I made during this years Mateus Braga, David Rosa, Willmar Seródio, Marcelo Pires, Gabriel Soares, Octavio Junqueira, Anderson Tomaz and Guilherme Sadovski besides the old one Vitor Leandro. You can be sure you helped me with your camaraderie and friendship. I am also thankful for the friendship and support of my friends from Espírito Santo, especially Luan and Marcio. I am thankful to the entire Schunck family, which is my second family for the love and support. I am especially thankful to my love Amanda that, even far away nowadays, was on my side during the last nine years. I would also like to thank to all the professors that accepted to be part of my doctoral board. I would like to thank my parents for raising me and giving me all the love and support I needed to pursue my educational goals and my dreams. I am grateful to CNPq for financial support along the last four years.

Dedicated to the memory of my father, Victor Marcos de Lima Ferreira. The one who always cheered for me and was always there for me when I needed him. Love and miss you, dad.

Abstract

This thesis intends to explore various methods for obtaining intricate insights into the behavior of Yang-Mills theory in the infrared regime. The primary focus lies on two key aspects: the Abelian dominance conjecture and the gluonic phase transition between confined and deconfined regimes.

Three distinct paths were followed to achieve these objectives.

The first path involved investigating the Yang-Mills-Chern-Simons theory as an effective 3-dimensional field theory that describes the 3-dimensional Yang-Mills-Higgs model. This model, in turn, serves as an effective field theory for the 4-dimensional Yang-Mills theory at high temperatures. By incorporating the horizon function to eliminate infinitesimal Gribov copies, a novel massive term called the Gribov parameter was introduced. Its determination through the gap equation modified the pole structure of the gluon propagator and led to the emergence of complex mass degrees of freedom, indicative of confinement in the infrared regime. Additionally, by restricting the path integral to the Gribov region, all finite Gribov copies were also eliminated, resulting in a Gribov copy-free path integral. Notably, the quantization rule imposed to ensure gauge invariance of the Chern-Simons action revealed novel findings not obtained in previous works.

The second path focused on the Refined Gribov-Zwanziger model at finite temperature. Through a semi-classical approach, an effective

massive Yang-Mills model incorporating thermal effects was introduced. The study explored the effects of the Gribov parameter and 2-dimensional condensate masses on the pole structure of propagators and phase transitions. Two critical temperatures for phase transitions were identified, and it was demonstrated that condensate masses decrease these transition temperatures. The discovery of a partial confinement phase aligned with previous research on quark-gluon plasma transitions. Future research plans include extending the method to 1-loop order and analyzing condensate masses at finite temperatures, as well as examining longitudinal and transverse masses.

The third path involved constructing an effective massive Yang-Mills model within the maximal Abelian gauge to investigate the infrared regime of Yang-Mills theory. Two massive terms encapsulating non-perturbative information were introduced. The gluon self-energy of both the Abelian and non-Abelian sectors were calculated at 1-loop, enabling the determination of the Abelian and non-Abelian effective masses. Comparisons with lattice results were made, confirming the model's agreement with lattice simulations and providing evidence of Abelian dominance.

The research culminated in the publication of two articles analyzing the Yang-Mills-Chern-Simons theory and exploring the maximal Abelian gauge. Other two articles studying the infrared behaviour of the massive Yang-Mills model and of the Yang-Mills theory within the Gribov-Zwanziger framework are currently in progress. The findings contribute to a deeper understanding of the infrared behavior of Yang-Mills theory and shed light on the intricate dynamics of confinement and phase transitions within the theory.

Abstract

Esta tese tem como objetivo explorar diversos métodos para obter uma compreensão mais aprofundada do comportamento da teoria de Yang-Mills no regime infravermelho. O foco principal está em dois aspectos fundamentais: a conjectura da dominância Abelian e a transição de fase gluônica entre os regimes confinado e não confinado.

Foram seguidos três caminhos distintos para alcançar esses objetivos.

O primeiro caminho envolveu a investigação da teoria de Yang-Mills-Chern-Simons como uma teoria efetiva de campo tridimensional que descreve o modelo de Yang-Mills-Higgs tridimensional. Esse modelo, por sua vez, serve como uma teoria efetiva de campo para a teoria de Yang-Mills quatro-dimensional em altas temperaturas. Ao incorporar a função horizonte para eliminar as cópias infinitesimais de Gribov, um novo termo massivo chamado de parâmetro de Gribov foi introduzido. Sua determinação através da equação de gap modificou a estrutura de pólos do propagador do glúon e levou à emergência de graus de liberdade de massa complexa, indicativos de confinamento no regime infravermelho. Além disso, restringindo a integral de caminho à região de Gribov, todas as cópias finitas de Gribov também foram eliminadas, resultando em uma integral de caminho livre de cópias de Gribov. Notavelmente, a regra de quantização imposta para garantir a invariância de calibre da ação de Chern-Simons revelou descobertas inéditas não obtidas em trabalhos anteriores.

O segundo caminho concentrou-se no modelo de Gribov-Zwanziger refinado a temperatura finita. Através de uma abordagem semiclássica, um modelo efetivo de Yang-Mills massivo incorporando efeitos térmicos foi introduzido. O estudo explorou os efeitos do parâmetro de Gribov e das massas de condensados bidimensionais na estrutura de pólos de propagadores e nas transições de fase. Foram identificadas duas temperaturas críticas para as transições de fase e demonstrou-se que as massas dos condensados diminuem essas temperaturas de transição. A descoberta de uma fase de confinamento parcial está alinhada com pesquisas anteriores sobre as transições do plasma de quarks-glúons. Os planos futuros de pesquisa incluem estender o método para a ordem de 1-loop e analisar as massas dos condensados em temperaturas finitas, além de examinar as massas longitudinais e transversais.

O terceiro caminho envolveu a construção de um modelo efetivo de Yang-Mills massivo dentro da calibre maximal Abelian para investigar o regime infravermelho da teoria de Yang-Mills. Dois termos massivos encapsulando informações não perturbativas foram introduzidos. A autoenergia do glúon nos setores Abelianos e não-Abelianos foi calculada na ordem de 1-loop, permitindo a determinação das massas efetivas Abelianas e não-Abelianas. Foram feitas comparações com resultados de lattice, confirmando a concordância do modelo com as simulações de lattice e fornecendo evidências de dominação Abelianas.

A pesquisa culminou na publicação de dois artigos que analisam a teoria de Yang-Mills-Chern-Simons e exploram a calibre maximal Abelian. Outros dois artigos que estudam o comportamento infravermelho do modelo de Yang-Mills massivo e da teoria de Yang-Mills dentro do framework Gribov-Zwanziger estão atualmente em andamento. As descobertas contribuem para uma compreensão mais profunda do comportamento infravermelho da teoria de Yang-Mills e lançam luz sobre a dinâmica intricada de confinamento e transições de fase dentro da teoria.

Contents

List of Figures

Introduction

The confinement of quarks and gluons represent an open problem for the theoretical physics community. It is an experimentally well documented phenomenon, consisting on the fact that 'color-charged' particles are never found in completely free states, but always bounded with other ones in order to form 'colorless' states. To describe such phenomenon would mean in principle to understand the transition between the confined and deconfined regimes of the Yang-Mills theory [5][6][7]. With this in mind, the main goal of this thesis is exactly to analyze the phase transition from the confined and deconfined regimes of the Yang-Mills (YM) theories and of some effective massive models that can be used in order to describe the Quantum Chromodynamics (QCD) in some of its regimes.

We will analyse such phase transitions within the following three different contexts:

- The Yang-Mills-Chern-Simons model within the Gribov-Zwanziger framework (at chapter 3);
- The Yang-Mills theory within the Gribov-Zwanziger model at finite temperatures (at chapter 4);
- The massive Yang-Mills theory (at chapter 5).

In all of these works, the main goal is always to calculate the gluon two-point function of the model at tree-level and/or at 1-loop and then to analyse its poles structure and its form factor. From the poles structure we will be able to observe what we will consider as a phase transition between the confined and deconfined regimes of the theory. To be more precise, we will consider as deconfined all the degrees of freedom to which we can associate a real positive mass, while the confined ones will be all the degrees of freedom with negative or complex masses. We will do so, due to the fact that if a particle is confined it meas that it necessarily is out from the physical spectrum, meaning that its mass cannot be measured.

This thesis is stuctured in the following form,

- In the first chapter we will make a brief revision of the Yang-Mills theory, revisiting the path integral quantization method, as well as the Faddeev-Popov gauge fixing procedure. We will also take advantage to talk about the Landau gauge, the linear covariant gauges and the maximal Abelian gauge, which are the gauges upon all this work is established.

- In the second chapter we will make a brief overview about the Gribov problem and how we circumvent it by means of the Gribov-Zwanziger method. At the end of the chapter we will also introduce the refined Gribov-Zwanziger model in which we introduce effective terms in the action of the theory that take into account the effects of the condensation of 2-dimensional operators, modifying the non-perturbative poles structure of the gluon propagators.

- In the third chapter we will talk about the 3-dimensional Euclidean Yang-Mills-Chern-Simons (YMCS) theory whithin the Gribov-Zwanziger framework both in the linear covariant gauges and in the maximal Abelian ones. Such model is defined by the addition to the Yang-Mills action of a topological Chern-Simons term which, as can be seen at [8], adds a massive parameter to the poles of the bosonic gauge field propagators that is of purely topological origin.

This Euclidean 3-dimensional model is important due to its connection with the Yang-Mills-Higgs theory in the Minkowski space-time, which is an effective model for the Quantum Chromodynamics at high temperatures, what can be seen at [9], where it is shown that at such regimes the fermionic (quarks) fields decouple from the bosonic (gluons) ones due to the asymptotic freedom feature of the strong interaction [10][11][12], giving rise to a plasma of quarks and gluons. Besides having a completely different mass generation mechanism, such theories are profoundly connected to the Yang-Mills-Chern-Simons ones [9][13], since the gluon propagators of both theories manifest a qualitatively analogous behaviour both at the ultraviolet (deconfined) and the infrared (confined) regimes manifesting the same number of physical and unphysical degrees of freedom.

- In the fourth chapter we will talk about the Yang-Mills theory at finite temperatures whithin the Gribov-Zwanziger panorama in the linear covariant gauges. In this part of the work the idea is to calculate the thermal mass of the gluon propagators in terms of the longitudinal and transverse sectors of the 1-loop gluon self-energy. With it we build an effective action that leads the tree-level gluon propagators of the theory to gain such thermal mass contribution in its poles structure. Then, together with the evaluation of the gap equation of the theory we analyse the phase transition of the theory as well as the Gribov parameter influence in the process.

- In the fifth chapter we study the $SU(2)$ massive Yang-Mills theory in the maximal Abelian gauge at 1-loop. In this part of the work the idea is to condense all of the non-perturbative features of the theory inside the effective Abelian and non-Abelian effective masses that we added in the theory. Then, we calculated the gluon Abelian and non-Abelian self-energies of the model and, with them, we obtained the Abelian and non-Abelian gluon propagators with their corresponding 1-loop corrections. After that we fitted our effective masses with previous lattice calculations in order to obtain their values. We concluded observing that the non-Abelian mass of the theory is bigger that the Abelian one, which is good agreement with the Abelian dominance conjecture.

Chapter 1

The Yang-Mills Theories

Summary

Over the last century, much effort has been put into unraveling the intricacies of our universe. This pursuit was greatly accelerated by the advent of Quantum Field Theory (QFT) which brought about a major revolution in the field of physics. Such progress brought new methods for calculating particle behavior, such as the Feynman diagram and the Schwinger-Dyson equation and led to the development of the Quantum electrodynamics (QED), which was developed during the 1940s and the 1950s. By these methods, precise predictions of particle behavior in high-energy collisions were made possible. During the 1960s and 1970s, the electroweak theory and Quantum chromodynamics (QCD), which describe the electromagnetic and the strong and weak nuclear forces, were developed by physicists and incorporated to the standard model.

The theory of Quantum Electrodynamics (QED) is characterized as a gauge theory because it is invariant under $U(1)$ gauge transformations. However, QED is just a special (Abelian) case of the more general non-Abelian gauge theories, known as Yang-Mills theories (YM), which are invariant under $SU(N)$ gauge

transformations. Such $SU(N)$ symmetry group has a particular physical interest since it is the symmetry group of the standard model, in particular, the symmetry group of the QCD. These theories were developed by Yang and Mills [5] and describe three of the four fundamental interactions: electromagnetic, nuclear weak, and strong interactions. However, we must emphasise that the YM theories describe only the pure gauge sector of the standard model, needing to be supplemented with by fermions and by the Higgs boson in order to represent all the particles of the standard model and their interactions.

This thesis aims to study non-perturbative aspects of Yang-Mills theories [10; 11] at both zero and finite temperatures. This will be done mainly through two approaches: the use of effective models such as Yang-Mills-Chern-Simons and massive Yang-Mills-Theories, and the use of the Refined-Gribov-Zwanziger quantization method [14; 15].

Before diving into Yang-Mills theories, it is important to understand what a gauge theory is. A gauge theory is a theory whose Lagrangian is invariant under gauge transformations, which are local transformations on the transformation parameter. These transformations are dependent on the position and are defined by,

$$A_\mu \to A'_\mu = u^{-1}(x)\left(A_\mu + \frac{i}{g}\partial_\mu\right)u(x), \tag{1.1}$$

where g is the coupling constant, A_μ is the gauge field and $u(x)$ is a unitary $N \times N$ complex matrix satisfying $u^\dagger = u^{-1}$ and $det(u) = 1$. The Lagrangian of a gauge theory is invariant under the action of a local element of a Lie group, while the gauge fields take values in the Lie algebra.

Additionally, all physically observable quantities in Nature are expected to be gauge invariant, meaning that the fields A_μ are not physical fields. This represents the gauge principle, the fundamental basis of gauge theories. Thus, the set of all possible physical transformations that leave invariat the physical observables of a physical system define a group which is known as the gauge group.

The particular gauge theory that is being considered will determine, to some extent, the nature of the gauge group. There are two types of gauge group; the Abelian ones and the non-Abelian ones. A group is said to be Abelian gauge when the components of the group commute with each other and the group itself, which means that the gauge transformations at various positions in space and time commute with each other. An example of Abelian gauge group is the $U(1)$ group, which describes a continuous symmetry of the electromagnetic field under local phase transformations.

Yet, a non-Abelian gauge group is a group where the components of the group do not commute with each other. A group being non-Abelian basically means that the gauge transformations at distinct positions in space and time do not commute with one other. An example of a non-Abelian gauge group is the $SU(3)$ group, which which describes the strong nuclear force between quarks and gluons.

1.1 The Abelian case

As previously said, the most notorious example of an Abelian gauge theory is the $U(1)$ gauge theory, which describes the electromagnetic theory. Its pure gauge sector Lagrangian is invariant under the following transformations of the gauge fields,

$$A_\mu \to A_\mu + \frac{i}{e} e^{-\xi(x)} \partial_\mu e^{\xi(x)}, \tag{1.2}$$

with e being the electric charge. Such fields take values in the algebra of U(1) (with $\xi(x)$ being one of its elements) and transform, as on eq.(1.1), under the action of an element $e^{\xi(x)}$ of this gauge group, whose Abelian nature is manifest by the following relation,

$$[A_\mu, A_\nu] = 0. \tag{1.3}$$

From the field A_μ we can also define the covariant derivative D_μ of the theory, which accounts for changes in the basis of a vector or tensor field under parallel transport along a curved manifold and it is given, in this case by,

$$D_\mu = \partial_\mu - ieA_\mu. \tag{1.4}$$

Then, as the electromagnetic strength tensor $F_{\mu\nu}$ is defined in terms of the commutator of the covariant derivatives D_μ of the theory, we have that,

$$F_{\mu\nu} = \frac{i}{e}[D_\mu, D_\nu]. \tag{1.5}$$

which can be also written as,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{1.6}$$

where eq.(1.3) was used.

Now, one is able to construct a Lagrangian density for the gauge sector of the electromagnetic theory, in order to study the dynamics of the gauge field A_μ. We have that the simplest gauge invariant and polynomial term involving the dynamical field and its derivatives forming a scalar, as required since the Lagrangian density is a scalar, is,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu}. \tag{1.7}$$

Since from such Lagrangian density we are able to derive the Maxwell eqs. by means of the Euler-Lagrange eqs. [16] we have a good justification to consider this choice of Lagrangian density for the pure gauge sector of the electromagnetic theory.

1.2 The non-Abelian case

In the previous section, we have talked about the Electromagnetic theory, that is an Abelian gauge theory. Now we will study a non-Abelian extension of it or, in other words, a model where the eq.(1.3) is not fulfilled, leading to the Yang-Mills theories.

The Yang-Mills theories are a class of non-Abelian gauge theories whose symmetry group is the $SU(N)$. Here the Yang-Mills fields $A_\mu(x)$ assume values in the Lie algebra of the group $SU(N)$ and, in the adjoint representation, can be written as matrices,

$$A_\mu(x) = \sum_{a=1}^{N^2-1} A_\mu^a T^a \equiv A_\mu^a T^a, \tag{1.8}$$

where we have used Einstein's summation convention, suppressing the sum symbol which runs over the $N^2 - 1$ group generators T^a, with index group a, what means that we have $N^2 - 1$ gauge bosons A_μ^a that intermediate the Yang-Mills interactions.

The generators T^a of the $SU(N)$ group are $N \times N$ Hermitian matrices with vanishing trace that form an ortonormal basis and obey the relations,

$$\begin{aligned} [T^a, T^b] &= i f^{abc} T^c \\ tr(T^a T^b) &= \frac{1}{2}\delta^{ab}, \end{aligned} \tag{1.9}$$

where f^{abc} is the group structure constant, which obeys the Jacobi identity,

$$f^{abc}f^{cde} + f^{adc}f^{ceb} + f^{aec}f^{cbd} = 0. \tag{1.10}$$

From eq.(1.1) we can write the infinitesimal gauge transformations of the Yang-Mills theories as,

$$A_\mu^a \to A_\mu^a + D_\mu^{ab}\xi^b, \tag{1.11}$$

which are non-linear transformations where,

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - gf^{abc}A_\mu^c, \tag{1.12}$$

is the covariant derivative in the adjoint representation of the gauge group and g is the coupling constant of the theory. It can be generically written in the fundamental representation of the gauge group as,

$$D_\mu = \partial_\mu + ig[A_\mu,]. \tag{1.13}$$

Thanks to the non-Abelian feature of the Yang-Mills fields, we have that,

$$[A_\mu, A_\nu] = [A_\mu^a T^a, A_\nu^b T^b] = if^{abc}A_\mu^a A_\nu^b T^c. \tag{1.14}$$

So, analogous to the previous case, we have that the Yang-Mills strength tensor will be given by,

$$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu], \tag{1.15}$$

being able to be rewritten by means of eq.(1.12) as,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu. \tag{1.16}$$

From the strength tensor we have that the Yang-Mills Lagrangian density is,

$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \tag{1.17}$$

and, consequently, the Yang-Mills action will be,

$$S_{YM} = -\int d^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right). \tag{1.18}$$

As it can be seen, one important difference between the electromagnetic theory and the Yang-Mills one is that the non-Abelian term $g f^{abc} A^b_\mu A^c_\nu$. Obviously this non-Abelian feature has physical consequences. First of all, for such theories the non-Abelian bosonic gauge field A_μ carries the charge of the interaction which it intermediates, which means that such fields interact with each other, while the electromagnetic theory do not manifest this kind of feature. This property finds out a parallel in General Relativity [17], where the gravitational field also carries energy/momentum, which is its source, so that the field becomes also a source, interacting with itself.

1.3 Quantization of gauge theories

The quantization of gauge theories can be achieved by Feynman's path integral approach, which is described in various references [18; 19; 20; 21]. This method starts with defining the partition function, also known as the generating functional, Z as,

$$Z = \frac{1}{N} \int DA \, e^{-S[A]}, \tag{1.19}$$

where $S[A]$ is the action of the gauge theory.

In this equation, the gauge field $A_\mu(x)$ is integrated over all possible configurations. This is represented by the functional measure DA. The action S and the normalization constant N are also included.

As it was shown on eq. (1.1), due to the gauge symmetry, for each gauge field configuration A_μ there will always be an infinity of other physically equivalent gauge field configurations A'_μ that will be related to it and to each other by some gauge transformation. A set of physically equivalent[1] gauge field configurations is referred as a 'gauge orbit'.All the gauge orbits, which fill the gauge field configuration space, can be thought of as smooth, non-intersecting trajectories inside of such space (see figure 1.1). Every point along one of these trajectories is a gauge field configuration, and all points are related to each other through gauge transformations. It is important to note that all gauge field configurations belong to a gauge orbit, and gauge fields in different gauge orbits are not related to each other by a gauge transformation.

Because of this, when we perform the integration over all the gauge field configurations A_μ, we end up making an overcounting of all physically equivalent gauge field configurations. Such a thing leads to the divergence of the path integral of eq.(1.19).

Another problem that arises is the fact that, in gauge theories, due to the gauge symmetry, we cannot obtain the gauge propagator of the theory since it is non-invertible.

[1]By 'physically equivalent' configurations we mean the gauge field configurations related to each other by a gauge transformation.

In order to properly solve these problems we are forced to fix the gauge (see figure 1.1), in such a way we only sum, in the path integral, over one representative of each gauge orbit. This means basically that we are led to select a surface in the gauge field configuration space which crosses all the gauge orbits once, and only once, choosing, in such a way, the configurations that will be the representative of each orbit and over which we will sum in the path integral.

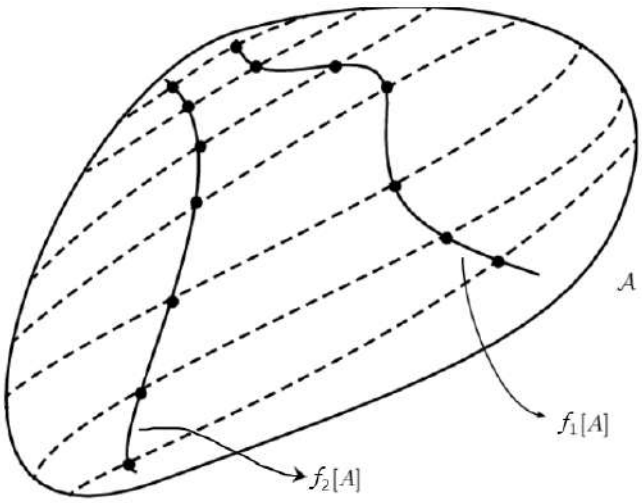


Figure 1.1: This is a section of the gauge field configuration space $\mathcal{A}$. Here the dashed lines represent the gauge orbits, while the full lines represent the transverse section of two different gauge fixing surfaces $f_1[A]$ and $f_2[A]$, as can be seen on [1].

The procedure through which we implement this gauge fixing is called the 'Faddeev-Popov quantization' [22][19]. To understand how it works, we first perform a gauge transformation over the gauge fields, given by eq. (1.1).

This leads the path integral of eq.(1.19) to become,

$$Z = \frac{1}{N}\int DuDA^u \; e^{-S[A^u]}det\left|\frac{\delta f[A^u(x)]}{\delta u(x')}\right|, \tag{1.20}$$

where $u(x)$ is an element of a Lie group that perform the gauge transformation to a gauge of our choice, Du it's the volume element of the group space, $det\left|\frac{\delta f[A^u(x)]}{\delta u(x')}\right|$ is the Jacobian of this transformation and $f[A^u(x)]$ is the function that characterize our gauge choice. In order to simplify our notation, we will write the Jacobian as,

$$det\left|\frac{\delta f[A^u(x)]}{\delta u(x')}\right| \equiv det(-M), \tag{1.21}$$

with M being the Faddeev-Popov operator.

Now, after having made the gauge transformation to a gauge of our choice, we still need to ensure that the path integral will only sum over gauge inequivalent fields A_μ. To do so, we can add to the path integral a Dirac delta of the form,

$$\delta(f[A^u(x)]), \tag{1.22}$$

which is responsible to implement the gauge fixing condition,

$$f[A^u(x)] = 0, \tag{1.23}$$

leading the path integral (1.20) to become,

$$Z = \frac{1}{N}\int DuDA^u\ e^{-S[A^u]}det(-M)\delta(f[A^u]). \tag{1.24}$$

One can easily see at [19] that,

$$\int Du\delta(f[A^u(x)])det(-M) = 1, \tag{1.25}$$

what when substituted in eq.(1.24) we obtain that,

$$Z = \frac{1}{N}\int DA^u\ e^{-S[A^u]}, \tag{1.26}$$

where DA^u and $S[A^u]$ are gauge invariant quantities, leading us back to eq.(1.19), which means that the partition function is a gauge invariant quantity. This is good since we directly calculate physical observables from it, so it must, in fact, conform with the gauge principle. In other words, we showed that, by fixing the gauge into the action by introducing a Dirac delta function into the path integral, which in principle breaks the gauge simmetry of the action, it still does not affect the gauge invariance of the path integral, meaning that its physical content is left unchanged.

Another thing that must be perceived is that on eq.(1.24), due to the gauge invariance of the terms DA^u, $e^{-S[A^u]}$, $det(-M)$ and $\delta(f[A^u])$ we can write it in the following form,

$$\begin{aligned} Z &= \frac{1}{N}\int DA\, e^{-S[A]}det(-M)\int Du\delta(f[A^u]) \\ &= \frac{1}{N}\int DA\, e^{-S[A]}det(-M)\delta(f[A])\int Du \\ &= \frac{1}{N}\int DA\, e^{-S[A]}det(-M)\delta(f[A]) \end{aligned} \tag{1.27}$$

where we can see that the integral $\int Du$ can be separated out from the path integral, meaning that it can be absorbed by the normalization constant.

Now, we are also able to transform the gauge fixing condition by adding an arbitrary function $F(x)$, in the form,

$$f[A(x)] \to f[A(x)] - F(x), \tag{1.28}$$

which is just a parametrization of the gauge fixing, leading eq.(1.27) to become,

$$Z = \frac{1}{N}\int DAe^{-S[A]}det(-M)\delta(f[A(x)] - F(x)). \tag{1.29}$$

Now averaging the path integral over this arbitrary function $F(x)$ with a Gaussian weight properly normalized to unity of the form,

$$e^{-\int \frac{F^2(x)}{2\alpha} d^4x}, \tag{1.30}$$

where α is a gauge parameter. This makes the quadratic part of the action of non-Abelian gauge theories to become non-degenerate, we find that the path integral (1.29) becomes,

$$\begin{aligned} Z &= \frac{1}{N(\alpha)} \int DC\ Ze^{-\int \frac{F^2(x)}{2\alpha} d^4x} \\ &= \frac{1}{NN(\alpha)} \int DCDA\ e^{-S[A]-\int \frac{F^2(x)}{2\alpha} d^4x} det(-M)\delta(f[A(x)] - F(x)) \\ &= \frac{1}{N'} \int DA\ e^{-S[A]-\int \frac{f^2[A(x)]}{2\alpha} d^4x} det(-M) \\ &= \frac{1}{N'} \int DA\ e^{-S[A]-S_{GF}[A]} det(-M), \end{aligned} \tag{1.31}$$

where $N' = NN(\alpha)$ is the normalization constant, α is an arbitrary constant and,

$$S_{GF} = \int \frac{f^2[A(x)]}{2\alpha} d^4x \tag{1.32}$$

is the gauge fixing action.

As can be seen in [19], the matrix M can be written in terms of an integral over Grassmanian variables $\bar{c}$ and c, as follows,

$$det(-M) = \int D\bar{c}Dc\ e^{\int d^4x(\bar{c}^a M^{ab} c^b)}, \tag{1.33}$$

where we written it in the adjoint representation. Here M^{ab} is the Faddeev-Popov operator and the c^b and $\bar{c}^a$ are Grassmann fields which in this case are called respectively as the Faddeev-Popov ghost and anti-ghost fields, that are scalar fields that obey anti-commutation relations. This implies their field excitations are causality-violating particles, not belonging to the physical spectrum or, in other words, virtual particles, appearing in the Feynman diagrams only as internal loops and never having asymptotic states [19; 20]. However, such fields are important because they cancel non-physical degrees of freedom of the gauge fields playing a pivotal role to make the theory unitary [20]. The term $S_{FP} = -\int d^4x(\bar{c}Mc)$ will be called as the Faddeev-Popov action.

Substituting eq.(1.33) into eq.(1.31) we obtain,

$$\begin{aligned} Z &= \frac{1}{N'}\int [DA][D\bar{c}][Dc]\; e^{-S[A]-S_{GF}[A]+\int d^4x(\bar{c}^a M^{ab} c^b)} \\ &= \frac{1}{N'}\int [DA][D\bar{c}][Dc]\; e^{-S[A]-S_{GF}[A]-S_{FP}[A]} \\ &= \frac{1}{N'}\int [DA][D\bar{c}][Dc]\; e^{-S_{eff}[A]}, \end{aligned} \tag{1.34}$$

with $S_{eff} = S + S_{GF} + S_{FP}$ being the effective action of the theory.

As can be seen on [22; 23; 24; 25], another way of writing the gauge fixing action can be,

$$S_{GF} = \int d^4x \left(b^a(x) f^a(x) + \frac{\alpha}{2} b^a b^a \right), \tag{1.35}$$

where $b^a(x)$ is the Lautrup-Nakanishi field, which is a Lagrange multiplier for the coupling we are inserting in the theory, the gauge condition.

We can observe form eq.(1.34) that it manifests a symmetry called as 'Faddeev-Popov symmetry', which is characterized by the invariance under the following transformations over the ghost fields,

$$\begin{aligned} c^a &\to e^{i\theta} c^a \\ \bar{c}^a &\to e^{-i\theta} \bar{c}^a, \end{aligned} \tag{1.36}$$

with θ being a finite, commuting and global parameter. Such symmetry leads to a new conserved charge called as 'ghost number', where the ghost and anti-ghost fields will respectively have, by convention, ghost numbers 1 and -1.

Another important feature of the Faddeev-Popov quantization procedure is that even if the gauge symmetry of the action is broken, there still exist a residual symmetry; the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [26; 27; 28; 29]. It is characterized by the fermionic operator s, which is a nilpotent ($s^2 = 0$) operator. It is a kind of supersymmetry, since it relates fermions to bosons and vice versa, being given by,

$$\begin{aligned} sA^a_\mu &= -D^{ab}_\mu c^b \\ sc^a &= \frac{g}{2} f^{abc} c^b c^c \\ s\bar{c}^a &= b^a \\ sb^a &= 0 \end{aligned} \tag{1.37}$$

Moreover, we have that,

$$sS_{eff} = 0. \tag{1.38}$$

It is important to observe that the BRST symmetry transformation has ghost number 1 and dimension 0, so it increases by 1 the ghost number of the field upon which it is applied, which is one of the many differences between the BRST and the conventional gauge symmetry [28; 29].

Another important feature of the BRST symmetry is that, from the nilpotency of the BRST operator s, we are able to define its cohomology [29; 30]. Thus taking an arbitrary BRST-invariant object Ψ^g_n, with dimension n and ghost number g, in such a way that it obeys the following equation,

$$s\Psi_n^g = 0, \tag{1.39}$$

we are able to rewrite Ψ as,

$$\Psi_n^g = C_n^g + sE_n^{g-1}, \tag{1.40}$$

where,

$$\begin{aligned} sC_n^g &= 0, \\ C_n^g &\neq s(something), \\ s^2 E_n^{g-1} &= 0. \end{aligned} \tag{1.41}$$

We call C_n^g as the closed, or non-trivial, sector of the BRST cohomology, while E_n^{g-1} is called as the exact, or trivial, sector of the BRST cohomology.

1.4 Linear Covariant gauges

Within the context of the Faddeev-Popov quantization we chose the Linear Covariant (LC) gauges as one of our gauge fixings to work with.The significance of the linear covariant gauges arises from their dual capacity to preserve the covariance of the theory and offer a practical framework for performing calculations. Covariance, in the context of gauge theories, pertains to the property wherein equations and physical observables maintain their structure under local gauge transformations. This property ensures the theory's consistency and mathematical coherence. Linear covariant gauges provide an effective means to maintain covariance while facilitating computational convenience. They serve as a valuable tool for performing calculations without compromising the essential properties of the theory.

One example of a linear and covariant gauge is the Landau gauge,

$$\partial_\mu A^a_\mu = 0, \tag{1.42}$$

which is a really manageable gauge fixing to work on, being for this reason a very popular one, having a great amount of papers on literature [31; 32; 33; 34; 35; 36] making use of it even on lattice [37; 38]. In such works, as well as the ones done directly in the linear covariant gauges, it were found evidences that the gluon propagator is suppressed in the infrared regime, while the ghost propagator is enhanced, which is something that is in agreement with the results obtained within the Refined-Gribov-Zwanziger model [14; 39].

The general case of linear covariant gauges is defined by the following condition,

$$\partial_\mu A^a_\mu = i\alpha b^a, \tag{1.43}$$

where α is the gauge parameter. It can be easily seen that the Landau gauge is recovered in the limit where the gauge parameter $\alpha \to 0$. Using the Faddeev-Popov quantization procedure that we exposed in the previous section we find out that in the Linear Covariant gauges the path integral (1.19) becomes,

$$Z = \frac{1}{N} \int [DA][D\bar{c}][Dc][Db] e^{-S_{YM} - S^{LC}_{FP}} \tag{1.44}$$

where S_{YM} is the Yang-Mills action (1.18) and,

$$\begin{aligned} S_{FP}^{LC} &= S_{GF}^{LC} + S_{FP} \\ &= \int d^4x \left(ib^a \partial_\mu A_\mu^a + \frac{\alpha}{2} b^a b^a - \bar{c}^a M^{ab} c^b \right), \end{aligned} \tag{1.45}$$

is the sum of gauge-fixing action with the Faddeev-Popov one, both in the LC gauges. Here we have that,

$$M^{ab} = -\partial_\mu D_\mu^{ab}, \tag{1.46}$$

is the Faddeev-Popov operator, and D_μ^{ab} is the covariant derivative in the adjoint representation defined on eq. (1.12).

So the gauge fixed Yang-Mills action in the linear covariant gauges becomes,

$$S = S_{YM} + S_{FP}^{LC}. \tag{1.47}$$

Therefore, from the path integral (1.44) we can find the propagators of the theory in the Euclidean space-time,

$$\begin{aligned} \langle A_\mu^a(k) A_\nu^b(-k) \rangle &= \left[\frac{1}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \alpha \frac{k_\mu k_\nu}{k^4} \right] \delta^{ab}, \\ \langle \bar{c}^a(k) c^b(-k) \rangle &= -\frac{\delta^{ab}}{k^2}. \end{aligned} \tag{1.48}$$

1.5 Maximal Abelian gauge (MAG)

Another gauge fixing condition that will be important for the work of this thesis is the maximal Abelian gauge. This gauge fixing was chosen by two main reasons; the first is because there are not much calculations in the literature made on this gauge; the second is the fact it is implementable on lattice, which means that we are able to compare our analytic results with the ones obtained on lattice [4; 40].

Besides that, the maximal Abelian gauge is important due to the fact that with it we are able to separate the non-Abelian and the Abelian sectors of the Yang-Mills theory enabling us to investigate the existence of evidences for the Abelian Dominance hypothesis [41], which will be better explained along this thesis.

Before introducing explicitly this gauge condition, we need to introduce the Cartan decomposition, which is defined by,

$$A_\mu^A T^A = A_\mu^a T^a + A_\mu^i T^i, \tag{1.49}$$

where the capital group indices A run in the set $1, 2, 3, ..., (N^2 - 1)$ of the $SU(N)$ group. Regarding the lower case group indices, we have that the ones with the letters $a, b, c, ...$, which will represent the non-Abelian (off-diagonal) components, will run in the set $1, 2, 3, ...N(N-1)$, while the ones with the letters $i, j, k...$, which will represent the Abelian (diagonal) components, will run in the set of all the other $N - 1$ components of the $SU(N)$ group. Therefore, T^a and T^i will be respectively the non-Abelian and the Abelian generators.

It is important to state that, regarding the maximal Abelian gauge, all the computations that will be addressed in this thesis will be restricted to the $SU(2)$ symmetry group. Therefore, in the $SU(2)$ Yang-Mills theory such generators will be,

$$\mathbf{T^1} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \mathbf{T^2} = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \mathbf{T^3} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1.50}$$

which means that they are proportional to the Pauli matrices in the adjoint representation. With such matrices we can form a decomposed algebra,

$$\begin{aligned}\left[T^i, T^j\right] &= 0 \\ \left[T^a, T^i\right] &= -if^{abi}T^b \\ \left[T^a, T^b\right] &= if^{abc}T^c + if^{abi}T^i. \end{aligned} \tag{1.51}$$

In the $SU(2)$ case, one has only 3 generators, where the ones with the index $a = 1, 2$ will be the non-Abelian generators and the one with the index 3 will be the one and only Abelian generator. Therefore,

$$\begin{aligned}\left[T^3, T^3\right] &= 0 \\ \left[T^a, T^3\right] &= -if^{ab3}T^b \\ \left[T^a, T^b\right] &= if^{ab3}T^3, \end{aligned} \tag{1.52}$$

where the f^{abc} term does not exist in this special case since we only have 2 non-Abelian components.

Now, we are able to define the Maximal Abelian gauge conditions, which are,

$$\mathcal{D}^{ab}_\mu A^b_\mu \equiv (\delta^{ab}\partial_\mu - gf^{abi}A^i_\mu)A^b_\mu = 0, \tag{1.53}$$

$$\partial_\mu A^i_\mu = 0, \tag{1.54}$$

The first gauge condition, eq.(1.53), is obtained by minimizing the following functional,

$$A^2_{MAG}[A] = \int d^4x A^a_\mu(x) A^a_\mu(x), \tag{1.55}$$

with respect to the gauge condition, see [42; 43]. The second gauge condition, eq.(1.54), is not obtained from any functional extremization, actually being related to the necessity of fixing a residual $U(1)^{N-1}$ symmetry. This infinitesimal gauge transformations for both sectors can be written in their explicit form as,

$$A^a_\mu \rightarrow A^a_\mu + D^{ab}_\mu \omega^b + g f^{abc} A^b_\mu \omega^c + g f^{abi} A^b_\mu \omega^i, \tag{1.56}$$

$$A^i_\mu \rightarrow A^i_\mu + \partial_\mu \omega^i + f^{abi} A^a_\mu \omega^b, \tag{1.57}$$

with ω^i and ω^a being the components of the infinitesimal gauge transformation,

$$A^A_\mu \rightarrow A^A_\mu + \frac{1}{ig} \mathcal{D}^{AB}_\mu \omega^A, \tag{1.58}$$

in such a way that,

$$\omega^A T^A = \omega^a T^a + \omega^i T^i. \tag{1.59}$$

It is also important to expose that from eq.(1.10) we find out that the structure constants f^{abi} and f^{abc} obey the following relations,

$$f^{abc} f^{cde} + f^{abi} f^{ide} + f^{adc} f^{ceb} + f^{adi} f^{ieb} + f^{aec} f^{cbd} + f^{aei} f^{ibd} = 0, \tag{1.60}$$

$$f^{abc} f^{bdi} + f^{abd} f^{bic} + f^{abi} f^{bcd} = 0, \tag{1.61}$$

$$f^{abi} f^{bjc} + f^{abj} f^{bci} = 0. \tag{1.62}$$

After all those definitions, let us now consider the $SU(2)$ Yang-Mills theory, expressed within the Cartan decomposition, i.e., $A_\mu = A^A_\mu T^A = A^a_\mu T^a + A^3_\mu T^3$,

$$\begin{aligned} S_{YM} &= \frac{1}{4} \int d^4x F^A_{\mu\nu} F^A_{\mu\nu} \\ &= \frac{1}{4} \int d^4x \left(F^a_{\mu\nu} F^a_{\mu\nu} + F^3_{\mu\nu} F^3_{\mu\nu} \right), \end{aligned} \tag{1.63}$$

where the index $a = 1, 2$ stands for the non-Abelian components, and the index

3 stands for the Abelian one.

Using the Faddeev-Popov quantization procedure we find out that in the Maximal Abelian gauge the path integral (1.34) is,

$$Z = \mathcal{N} \int [DA^a][DA][D\bar{c}^a][Dc^a][D\bar{c}^3][Dc^3][Db^a][Db^3]e^{-S} \tag{1.64}$$

where,

$$S = S_{YM} + S_{FP}^{MAG}, \tag{1.65}$$

and,

$$S_{FP}^{MAG} = S_{GF}^{MAG} + S_{FP} \tag{1.66}$$

with,

$$\begin{aligned} S_{GF}^{MAG} &= \int d^4x \Bigg(ib^a \mathcal{D}_\mu^{ab} A_\mu^b + ib^3 \partial_\mu A_\mu^3 \Bigg), \\ S_{FP} &= \int d^4x \Bigg(- \bar{c}^a M^{ab} c^b + g f^{ab3} \bar{c}^a \left(\mathcal{D}_\mu^{bc} A_\mu^c \right) c^3 + \bar{c}^3 \partial_\mu \left(\partial_\mu c^3 + g f^{ab3} A_\mu^a c^b \right) \Bigg), \end{aligned} \tag{1.67}$$

being respectively the gauge fixing action and the Faddeev-Popov action in the MAG. Moreover we have that,

$$M^{ab} = -\mathcal{D}_\mu^{ac} \mathcal{D}_\mu^{cb} - g^2 f^{ac3} f^{bd3} A_\mu^c A_\mu^d, \tag{1.68}$$

is the Faddeev-Popov operator. This amounts to the implementation of the following conditions,

$$\begin{aligned} \partial_\mu A^3_\mu &= 0, \\ \mathcal{D}^{ab}_\mu A^b_\mu &= 0. \end{aligned} \tag{1.69}$$

From the path integral (1.64) we can find the propagators of the theory in the Euclidean space-time,

$$\begin{aligned} \langle A^3_\mu(k) A^3_\nu(-k)\rangle &= \frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\delta^{33}, \\ \langle A^a_\mu(k) A^b_\nu(-k)\rangle &= \frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}, \\ \langle \bar{c}^a(k) c^b(-k)\rangle &= -\frac{\delta^{ab}}{k^2}. \end{aligned} \tag{1.70}$$

Chapter 2

The Gribov Problem

2.1 Summary

In the previous chapter we discussed about the path integral quantization of gauge theories in the perturbative regime. However, in this chapter, we will discuss about the generalization of the path integral quantization of gauge theories to the non-pertubative regime. We will analyze the problems related to the arising of spurious gauge copies called 'Gribov ambiguities', and we will follow the steps made by Gribov [39; 44; 45; 46; 47] in order to deal with them.

As discussed in the previous chapter, the quantization of a gauge theory necessitates the gauge fixing of the generating functional at the action level. This crucial step addresses the issues of non-normalizability caused by gauge ambiguities and the non-invertibility of the quadratic gauge field operator. To accomplish this, we employed the Faddeev-Popov method, a powerful approach that incorporates a gauge fixing term into the effective action of the theory. Notably, the Faddeev-Popov term emerges from the Jacobian of the gauge transformation, adding an additional contribution to the overall dynamics of the system.

However, it is important to state that the entire Faddev-Popov quantization procedure is based upon two major hypothesis:

- The gauge fixing is always perfect, selecting only one representative of each gauge orbit.
- The Faddeev-Popov operator is always invertible.

Nevertheless, both hypothesis are not in general well justified, in such a way that the Faddeev-Popov procedure is not always valid, needing to be improved.

To understand the violation of the first hypothesis we must remember the explanation of the section 1.3 about the gauge fixing procedure. There we saw that to fix a gauge means to select a surface in the gauge field configuration space which crosses all the gauge orbits once, and only once, choosing, in such a way, the configurations that will be the representatives of each orbit and over which we will sum into the path integral. However, we have not emphasized that the gauge orbits are curves in the gauge field configuration space that can intersect the gauge fixing surface more than once, leading to the selection of more than one representative for the gauge orbits (see figure 2.1). This spurious extra configurations are called as 'Gribov ambiguities', or also, 'Gribov copies' and one of their effects on the Faddeev-Popov quantization procedure is that they make the path integral to become again non-normalizable, since we come back to integrate over equivalent gauge field configurations.

Concerning the second hypothesis we have that, in the Landau and Maximal Abelian gauges (where the calculations were already made), when the amplitudes of the gauge fields are sufficiently large or, in other words, when we enter in the non-perturbative regime, the operator M^{ab} will acquire zero modes (null eigenvalues), thus becoming non-invertible. This is also due to the presence of spurious new gauge copies (Gribov copies) in this regime, as we will see in the next section.

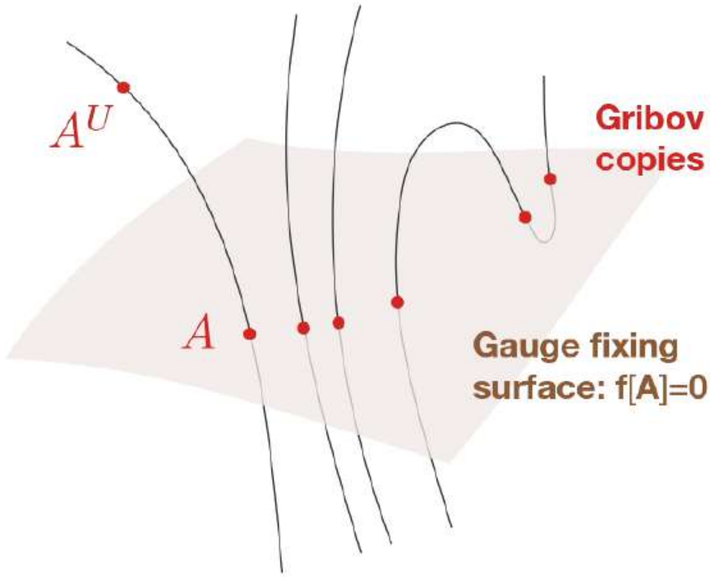


Figure 2.1: This is an heuristic representation of a gauge fixing surface intercepting multiple gauge orbits. We can see that since they are not straight lines but curves, the gauge fixing surface will end up intercepting them more then once. The original image can be found on [2]

Therefore, we can summarise the problem of the violation of the two major hypothesis of the Faddeev-Popov procedure as being part of one bigger problem that is known as 'the Gribov problem' [39; 44; 45; 46; 47].

2.2 The Gribov problem in the Landau gauge

As it was explained in the summary (2.1) the Faddeev-Popov quantization procedure works very weel in the perturbative regime of gauge theories. However, in the non-perturbative regime it is spoiled due to the arising of the Gribov ambiguities, which leads the Faddeev-Popov operator to manifest zero-modes, making it non-invertible [44; 45; 46; 47; 48].

One can easily understand this issue by analysing the Faddeev-Popov operator's eigenvalue equation in the Landau gauge, as follows,

$$\begin{aligned} M^{ab}\xi^{b} = -\partial_{\mu}D^{ab}_{\mu}\xi^{b} &= \varepsilon(A)\xi^{a}, \\ -\delta^{ab}\partial^{2}\xi^{b} + gf^{abc}\partial_{\mu}\left(A^{c}_{\mu}\xi^{b}\right) &= \varepsilon(A)\xi^{a}, \end{aligned} \tag{2.1}$$

where $\varepsilon(A)$ are the eigenvalues of M^{ab} and ξ^{a} being an infinitesimal gauge parameter of the gauge transformation $A'^{a}_{\mu} \to A^{a}_{\mu} - D^{ab}_{\mu}\xi^{b}$. One can see at eq.(2.1) that when the amplitude of the field A^{c}_{μ} is small, or when we turn off the coupling constant g, the first term $\delta^{ab}\partial^{2}\xi^{b}$ becomes dominant and leads to solution for ξ^{a} with positive eigenvalues $\varepsilon(A)$, meaning that at such regime the Faddeev-Popov operator is invertible. However, for sufficiently high values of gA_{μ} we can obtain negative eigenvalues for the Faddeev-Popov operator, meaning that, at some point, we necessarily reached its zero-modes. Moreover, the eigenvectors ξ^{a} with which these zero modes are assotiated will be the so-called 'infinitesimal Gribov copies', and,

$$-\partial_{\mu}D^{ab}_{\mu}\xi^{b} = 0 \tag{2.2}$$

will be known as the copies equation.

This means that, for high values of the coupling constant g (non-perturbative regime), a different approach [36; 45; 49; 50] might be implemented in order to circumvent the arising of infinitesimal Gribov copies and the zero-modes of the Faddeev-Popov operator with which they are assotiated.

A great progress was made by Zwanziger in [36; 45; 46; 49; 50; 51], where the Faddeev-Popov quantization procedure was improved by making a restriction on the domain of integration of the path integral (1.19) to a region of the gauge field configuration space where all the A^{a}_{μ} fields obey the gauge-fixing conditions and for which the Faddeev-Popov operator is invertible. This region is the so-called

Gribov region which presents important features like:

- It is bounded in all directions;
- It is convex;
- All the gauge orbits cross the Gribov region at least once.

The last property of the Gribov region is what guarantees that no physical information is lost in this restriction process. This is so due to the fact that all the gauge field configuration that lie outside Gribov region will be necessarily gauge copies (Gribov copies) of some configuration inside of it.

For the Landau gauge the Gribov region Ω will be defined as,

$$\Omega = \{A^a_\mu | \partial_\mu A^a_\mu = 0, M^{ab} > 0\}. \tag{2.3}$$

To implement such a restriction into the path integral (1.44) we need to impose the so-called 'no-pole condition'. In essence, we will impose that the ghost propagator does not develop poles, except the trivial one. As it is defined as the inverse of the Faddeev-Popov operator,

$$\langle \bar{c}^a(k) c^b(-k) \rangle = \langle k | (M^{-1}(A))^{ab} | k \rangle, \tag{2.4}$$

which, as can be seen on [45; 46; 52], up to one loop it is given by,

$$\langle \bar{c}^a(k) c^b(-k) \rangle \approx \frac{1}{k^2} \frac{1}{1 - \rho(k, A)} \tag{2.5}$$

where,

$$\rho(k, A) = \frac{N}{N^2 - 1} \frac{g^2}{V} \frac{k_\mu k_\nu}{k^2} \sum_q \frac{A^a_\mu(q) A^a_\nu(-q)}{(k - q)^2}, \tag{2.6}$$

and V is the volume of the Euclidean space. As it can be easily seen, the ghost propagator has two poles. The first one occurs at $k^2 = 0$, which is interpreted as the place where resides the Gribov horizon or, in other words, is the place where the first zero-mode of the Faddeev-Popov operator M^{ab}. Yet, when the second pole $\rho(k, A) = 1$ is reached, it means that we reached the second zero-mode of M^{ab}, meaning that we are necessarily outside the Gribov region. Therefore we conclude that for $\rho(k, A) < 1$ we would necessarily be inside the Gribov region. Moreover, as $\rho(k, A)$ is a monotonically decreasing function [46], we have that such a condition can be restricted to,

$$\rho(0, A) < 1. \tag{2.7}$$

Now, we can finally rebuilt our path integral (1.44) in the following form,

$$\begin{aligned} Z &= \mathcal{N} \int [DA][D\bar{c}][Dc][Db]\Theta(1 - \rho(0, A))e^{-S}, \\ &= \mathcal{N} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} \frac{d\zeta}{2\pi i\zeta} \int [DA][D\bar{c}][Dc][Db]e^{-S+\zeta(1-\rho(0,A))} \end{aligned} \tag{2.8}$$

with S given by eq.(1.47) and $\Theta(1 - \rho(0, A))$ being a Heaviside function that properly restricts the path integral to the Gribov region and that can be rewritten as an integral over ζ.

In order to analyse how such modification of the Faddeev-Popov method affects the gap equation [36; 39; 44; 45; 46; 47] and the two-point functions of the theory at the tree level, we only need to consider the quadratic terms over the fields on eq.(2.8), that will be given by,

$$\begin{aligned} Z^{quadr} &= \mathcal{N} \int \frac{d\zeta}{2\pi i} e^{f(\zeta)}, \\ f(\zeta) &= \zeta - ln\zeta - \frac{3}{2}(N^2 - 1) \sum_q ln \left(q^2 + \frac{Ng^2\zeta}{2Vq^2(N^2 - 1)} \right). \end{aligned} \tag{2.9}$$

Thus, this last integral can be evaluated by means of the steepest descent approximation method in order to find that,

$$Z^{quadr} \;=\; e^{f(\zeta_0)}, \tag{2.10}$$

where ζ_0 is the saddle point of the function $f(\zeta)$, meaning that,

$$f'(\zeta_0) = 0 \to 1 = \frac{1}{\zeta_0} + \frac{3}{4V}Ng^2\sum_q \frac{1}{q^4 + \frac{\zeta_0 Ng^2}{2V(N^2-1)}}. \tag{2.11}$$

Now, defining the so-called 'Gribov parameter' $\gamma^4 = \frac{\zeta_0}{4V(N^2-1)}$ we obtain that,

$$1 = \frac{1}{4V(N^2-1)\gamma^4} + \frac{3}{4V}Ng^2\sum_q \frac{1}{q^4 + 2Ng^2\gamma^4}. \tag{2.12}$$

So taking the thermodynamic limit of $V \to \infty$ we get,

$$1 = \frac{3}{4}Ng^2\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^4 + 2Ng^2\gamma^4}, \tag{2.13}$$

which is the so-called 'gap equation', from which we can determine the Gribov parameter γ^4 in terms of the coupling constant g and an ultraviolet cut-off Λ.

From eq.(2.8) one can also calculate the new gluon propagator, that will be given by,

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle = \frac{k^2}{k^4 + 2Ng^2\gamma^4}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}, \tag{2.14}$$

where it can be seen that it has a complex pole's structure, meaning that it is associated to non-physical degrees of freedom, since its mass will be out of the physical spectrum. This is an important feature since it allow us to interpret it as the propagator of confined gluons, since if a particle is confined, it might also be out of the physical spectrum of masses.

In order to continue to the next section, it must be stated that the restriction done to the path integral on eq.(2.8) was implemented by imposing the no-pole condition up to the first order of the ghost two-point function. However, an all order prescription was first made at [49], where instead of the no pole condition, it was only needed to impose that the minimum eigenvalue of the Faddeev-Popov operator had only positive values. In the Landau gauge, this means that the Gribov region will be defined by the condition,

$$\varepsilon_{min}(A) \geq 0. \tag{2.15}$$

Such a condition when applied to the trace of the Faddeev-Popov operator, expressed in eq.(1.68), leads to,

$$tr(M^{ab}(A)) = Vd(N^2-1) - H_L[A] > 0, \tag{2.16}$$

where $H_L[A]$ is the so-called 'horizon function', which properly restrict the Gribov region to all orders of the series expansion, and it is given by,

$$H_L[A] \;=\; \int d^4x d^4y \Bigg(g^2 f^{abc} A^b_\mu(x) [M^{-1}(A)]^{ad}(x,y) f^{dec} A^e_\mu(y) \Bigg). \tag{2.17}$$

After implementing the horizon function to the path integral, the total action becomes,

$$S_{GZ} \;=\; S_{YM} + S^L_{FP} + \int d^4x \Bigg(\gamma^4 H_L[A] + 4\gamma^4 V(N^2-1) \Bigg), \tag{2.18}$$

which is called as the Gribov-Zwanziger action. Here, the Gribov massive parameter γ^4 will be determined by the 'horizon condition', which is given by,

$$\langle H_L[A]\rangle = 4V(N^2-1). \tag{2.19}$$

As $H_L[A]$ is a non-local term, we can add some suitable auxiliary fields in order to properly localize it. This will be done by the following identity,

$$\begin{aligned} e^{-\gamma^4 H_L[A]} &= \int [D\bar{\phi}][D\phi][D\bar{\omega}][D\omega]exp\left[\int d^4x d^4y \left(\bar{\phi}^{ac}_\mu(x) M^{ab}(x,y)\phi^{bc}_\mu(y) + \right.\right. \\ &- \left.\left. \bar{\omega}^{ac}_\mu(x) M^{ab}(x,y)\omega^{bc}_\mu(y)\right) - g\gamma^2 \int d^4x f^{abc}A^a_\mu(x)\left(\bar{\phi}^{bc}_\mu(x)+\phi^{bc}_\mu(x)\right)\right], \end{aligned} \tag{2.20}$$

where $(\bar{\phi},\phi)$ and $(\bar{\omega},\omega)$ are, respectively, bosonic and fermionic pairs of auxiliary fields. This leads the path integral to assume the following form,

$$Z = \mathcal{N}\int [D\mu] e^{-S_{GZ}}, \tag{2.21}$$

where the $[D\mu] = [DA][D\bar{c}][Dc][Db][D\bar{\phi}][D\phi][D\bar{\omega}][D\omega]$ and,

$$\begin{aligned} S_{GZ} &= S_{YM} + S^L_{FP} - \int d^4x \left(\bar{\phi}^{ac}_\mu M^{ab}(A)\phi^{bc}_\mu - \bar{\omega}^{ac}_\mu M^{ab}(A)\omega^{bc}_\mu\right) + \\ &+ \int d^4x \left[g\gamma^2 f^{abc}A^a_\mu\left(\bar{\phi}^{bc}_\mu + \phi^{bc}_\mu\right) + 4\gamma^4 V(N^2-1)\right], \end{aligned} \tag{2.22}$$

is the new localized form of the Gribov-Zwanziger action in the Landau gauge.

2.2.1 The BRST symmetry of the Gribov-Zwanziger action in the Landau gauge

One important feature that can be analysed on eq. (2.22) is the BRST symmetry. As it can be seen on [20; 26; 27; 28; 29; 38; 47; 52; 53] the BRST symmetry of the GZ fields in the Landau gauge is given by,

$$
\begin{aligned}
sA_\mu^a &= -D_\mu^{ab} c^b \,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c \,, \\
s\bar{c}^a &= b^a \,, & sb^a &= 0 \,, \\
s\varphi_\mu^{ab} &= \omega_\mu^{ab} \,, & s\omega_\mu^{ab} &= 0 \,, \\
s\bar{\omega}_\mu^{ab} &= \bar{\varphi}_\mu^{ab} \,, & s\bar{\varphi}_\mu^{ab} &= 0 \,,
\end{aligned}
\tag{2.23}
$$

where s is the nilpotent BRST operator.

However, if we apply it on eq. (2.22) we find out that,

$$
sS_{GZ} \;=\; s \int d^4x \left[g\gamma^2 f^{abc} A_\mu^a \left(\bar{\phi}_\mu^{bc} + \phi_\mu^{bc} \right) \right] \neq 0, \tag{2.24}
$$

which means that the BRST symmetry is softly broken in the non-perturbative regime. We use the term 'softly' due to the fact that in the limit in the perturbative limit, which means $g \to 0$, or else $\gamma \to 0$, we reobtain the BRST symmetry.

Since the BRST symmetry is important to prove the renormalizability of the theory [29; 54], researchers tried some ways to reestablish it in the GZ model [52; 55; 56]. Then, in order to circumvent the problem of the BRST symmetry breaking in the non-perturbative regime, it was proposed on [55] to use a non-local, transverse, and gauge invariant field $A_\mu^{h,a}$, introduced in [57; 58]. Explicitly,

$A^{h,a}_\mu$ can be written as an infinite non-local series, given by,

$$A^h_\mu \;=\; \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + O(A^3)\right). \tag{2.25}$$

with A^h_μ being BRST invariant, since,

$$sA^h_\mu = 0. \tag{2.26}$$

With this non-local field we can rewrite the GZ action of eq. (2.22) in the form,

$$\begin{aligned} S_{GZ} \;&=\; S_{YM} + S^L_{FP} - \int d^4x \left(\bar{\phi}^{ac}_\mu M^{ab}(A^h)\phi^{bc}_\mu - \bar{\omega}^{ac}_\mu M^{ab}(A^h)\omega^{bc}_\mu\right) + \\ &+ \int d^4x \left[g\gamma^2 f^{abc} A^{h,a}_\mu \left(\bar{\phi}^{bc}_\mu + \phi^{bc}_\mu\right) + 4\gamma^4 V(N^2-1)\right], \end{aligned} \tag{2.27}$$

with,

$$M^{ab}(A^h) \;=\; -\delta^{ab}\partial^2 + g f^{abc} A^{h,a}_\mu \partial_\mu. \tag{2.28}$$

Even though we have the localizing auxiliary fields ϕ, $\bar{\phi}$, ω and $\bar{\omega}$, the action (2.27) is still non-local due to the presence of the non-local field A^h_μ. Thus in order to localize this new action, we add a new Stuekelberg field ξ^a as in [59]. Then we rewrite the field A^h_μ in a local fashion, in the form,

$$A^h_\mu = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h \tag{2.29}$$

with,

$$h = e^{ig\xi^a T^a}. \tag{2.30}$$

At last, we implement the transversality condition of the $A^{h,a}_\mu$ field,

$$\partial_\mu A^{h,a}_\mu = 0, \tag{2.31}$$

leading the action (2.27) to become,

$$\begin{aligned} S_{GZ} &= S_{YM} + S^L_{FP} - \int d^4x \left(\bar{\phi}^{ac}_\mu M^{ab}(A^h)\phi^{bc}_\mu - \bar{\omega}^{ac}_\mu M^{ab}(A^h)\omega^{bc}_\mu \right) + \\ &+ \int d^4x \left[g\gamma^2 f^{abc} A^{h,a}_\mu \left(\bar{\phi}^{bc}_\mu + \phi^{bc}_\mu \right) + 4\gamma^4 V(N^2-1) \right] + \\ &+ \int d^4x \left[\tau^a \partial_\mu A^{h,a}_\mu - \bar{\eta}^a M^{ab}(A^h)\eta^b \right], \end{aligned} \tag{2.32}$$

which is the local and BRST invariant version of the action (2.22). Here, τ^a is a Lagrange multiplier to ensure the transversality condition and the fields $(\bar{\eta}^a, \eta^a)$ are just ghost-like fields that come from the Jacobian that appears due to the implementation of such a restriction.

Finally, the complete set of BRST transformations that leaves the action

(2.32) invariant is

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,, \\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,, \\
s\varphi^{ab}_\mu &= 0\,, & s\omega^{ab}_\mu &= 0\,, \\
s\bar{\omega}^{ab}_\mu &= 0\,, & s\bar{\varphi}^{ab}_\mu &= 0\,, \\
sh^{ij} &= -igc^a (T^a)^{ik} h^{kj}\,, & sA^{h,a}_\mu &= 0\,, \\
s\tau^a &= 0\,, & s\bar{\eta}^a &= 0\,, \\
s\eta^a &= 0\,, & s\xi^a &= g^{ab}(\xi) c^b\,,
\end{aligned}
\tag{2.33}
$$

where,

$$
g^{ab}(\xi) = -\delta^{ab} + \frac{g}{2} f^{abc}\xi^c - \frac{g^2}{12} f^{amr} f^{mbq} \xi^q \xi^r + O(g^3)\,, \tag{2.34}
$$

and s is a nilpotent, i.e. $s^2 = 0$, BRST operator.

2.2.2 Extension to the linear covariant gauges

As it was previously seen on section 1.4, the Landau gauge, denoted by $\partial_\mu A^a_\mu = 0$, stands out as a special case ($\alpha = 0$) of the linear covariant gauges, denoted by $\partial_\mu A^a_\mu = -i\alpha b^a$. Thus, after obtaining a BRST invariant form of the GZ action in the Landau gauge, lets now obtain its generalization for the linear covariant gauges.

First of all we need to state that one important difference between the Landau and the linear covariant gauges is regarding the Hermicity property of the Faddeev-Popov operator $D^{ab}_\mu A^a_\mu$. This operator is Hermitian only for $\alpha = 0$, while for non-vanishing values of the parameter α it develops complex eigenvalues (meaning it is not Hermitian). Therefore such issue ends up spoiling the Gribov-Zwanziger analysis for the removal of infinitesimal Gribov copies (zero modes) from the path integral measure. However, thanks to the works of [55; 56] that gave us the basis for the formulation of a BRST-invariant GZ action in the Landau gauge, we are able now to overcome this problem in the linear covariant gauges also. The idea is basically the same of the section 2.2.1.

We will take the linear covariant action (1.45) and we will rewrite it in terms of the non-local and transverse gauge field A^h_μ, defined on eq. (2.25). Then we rewrite this field in a local fashion using eq. (2.29) where we introduce a Stuekelberg field ξ^a. Finally, we impose the transversality condition $\partial_\mu A^h_\mu = 0$ on the non-local field, leading us to the following action,

$$\begin{aligned} S_{GZ} &= S_{YM} + S^{LC}_{FP} - \int d^4x \left(\bar{\phi}^{ac}_\mu M^{ab}(A^h)\phi^{bc}_\mu - \bar{\omega}^{ac}_\mu M^{ab}(A^h)\omega^{bc}_\mu\right) + \\ &+ \int d^4x \left[g\gamma^2 f^{abc} A^{h,a}_\mu \left(\bar{\phi}^{bc}_\mu + \phi^{bc}_\mu\right) + 4\gamma^4 V(N^2-1)\right] + \\ &+ \int d^4x \left[\tau^a \partial_\mu A^{h,a}_\mu - \bar{\eta}^a M^{ab}(A^h)\eta^b\right], \end{aligned} \tag{2.35}$$

where S^{LC}_{FP} is the Faddeev-Popov action in the linear covariant gauges, given by eq. (1.45). So eq. (2.35) is the local and BRST invariant form of the GZ action in the linear covariant gauges. This means that with the action (2.35) we are implementing the restriction of the path integral to the Gribov region Ω, which will be given by

$$\Omega = \{A^a_\mu | \partial_\mu A^a_\mu = -i\alpha b^a, M^{ab}(A^h) > 0\}, \tag{2.36}$$

where $M^{ab}(A^h)$ is given on eq. (2.28).

From the action (2.35) we are able obtain the gluon propagator of the theory in the linear covariant gauges, which will be given by,

$$\langle A^a_\mu(k) A^b_\nu(-k)\rangle = \left[\frac{k^2}{k^4 + 2Ng^2\gamma^4}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \alpha\frac{k_\mu k_\nu}{k^4}\right]\delta^{ab}, \tag{2.37}$$

where it can be easily seen that in the limit $\alpha \to 0$ we recover the gluon propagator (2.14) of the Landau gauge.

2.3 The Gribov problem in the maximal Abelian gauge

As it was shown in the last section, the procedure of removing the infinitesimal Gribov copies form the path integral leads to a non-perturbative massive term in the gluon propagator in the Linear covariant gauges.

Now, we will repeat the same quantization procedure in the MAG in order to be able to analyse how the Abelian and non-Abelian sectors of the $SU(2)$ Yang-Mills theory is affected by the elimination of infinitesimal Gribov copies. First of all we derive the equation that defines the Gribov copies.

We start defining a gauge field $\bar{A}_\mu(x)$ whose Abelian and non-Abelian components respectively transform, as in eqs.(1.56) and (1.57), i.e.,

$$\bar{A}^3_\mu(x) = A^3_\mu - \partial_\mu \omega^3 - g f^{ab3} A^a_\mu \omega^b, \tag{2.38}$$

$$\bar{A}^a_\mu(x) = A^a_\mu - \mathcal{D}^{ab}_\mu \omega^b - g f^{ab3} A^b_\mu \omega^3. \tag{2.39}$$

Over the transformed fields of eqs. (2.38) an (2.39) we will impose the Maximal Abelian gauge conditions given by eqs. (1.53) and (1.54) in such a way that,

$$\partial_\mu \bar{A}^3_\mu = 0, \tag{2.40}$$

$$\mathcal{D}^{ab}_\mu(\bar{A}) \bar{A}^b_\mu = 0, \tag{2.41}$$

what will give us,

$$\partial_\mu A^3_\mu - \partial^2\omega^3 - gf^{ab3}\partial_\mu A^a_\mu \omega^b - gf^{ab3}A^a_\mu \partial_\mu \omega^b = 0, \tag{2.42}$$

$$\mathcal{D}^{ab}_\mu(\bar{A})A^b_\mu - \mathcal{D}^{ac}_\mu(\bar{A})\mathcal{D}^{cb}_\mu(A)\omega^b - gf^{ab3}\mathcal{D}^{bc}_\mu(\bar{A})(A^c_\mu\omega^3) = 0, \tag{2.43}$$

and so taking the first order approximation over the gauge parameters ω and ω^a and imposing again the gauge conditions over the equations above, we obtain,

$$-\partial^2\omega^3 - gf^{ab3}\partial_\mu A^a_\mu \omega^b - gf^{ab3}A^a_\mu\partial_\mu\omega^b \;=\; 0, \tag{2.44}$$

$$M^{ab}\omega^b \;=\; 0, \tag{2.45}$$

with,

$$M^{ab} = -\mathcal{D}^{ac}_\mu \mathcal{D}^{cb}_\mu - g^2 f^{ac3} f^{bd3} A^c_\mu A^d_\mu, \tag{2.46}$$

being the Hermitian, [60] non-Abelian Faddeev-Popov operator. The eqs. (2.44) and (2.45) give us the conditions for the existence of the Gribov copies. From eq. (2.44) we can see that the infinitesimal gauge parameter ω^3 is determined in terms of the parameter ω^a, as follows,

$$\omega^3 = -gf^{ab3}\frac{\partial_\mu}{\partial^2}(A^a_\mu\omega^b), \tag{2.47}$$

in such a way that redefining it by $\bar{\omega}^3 \to \omega^3 - gf^{ab3}\frac{\partial_\mu}{\partial^2}(A^a_\mu\omega^b)$ we obtain that,

$$\partial^2\bar{\omega}^3 = 0. \tag{2.48}$$

The parameter ω^a, in its turn, is determined by eq. (2.45). This means that the infinitesimal Gribov copies are essentially determined by eq. (2.45).

It is important to mention here that, as shown in [46], applying such transformation over the generating functional would lead to a field independent Jacobian, implying the decoupling of the Abelian ghosts of the theory and as a trivial consequence, they will not be affected by this generalization of the path integral quantization of gauge theories.

Now, we are finally able to start implementing the Gribov-Zwanziger approach in order to properly eliminate all the infinitesimal Gribov copies from the path integral (1.64). To do so, we apply on the path integral (1.64) the restriction to the Gribov region, which will be defined as,

$$\Omega_{MAG} := \{(A^3_\mu, A^a_\mu); \mathcal{D}^{ab}_\mu A^b_\mu = 0, \partial_\mu A^3_\mu = 0 | M^{ab} > 0\}. \tag{2.49}$$

Some of the properties of this region are,

- It is unbounded in all Abelian directions and bounded by the Gribov horizon in the non-Abelian ones as we can see in the figure 2.2;
- It is convex, meaning that any straight line drawn through an arbitrary point on this surface, in any direction, will intersect at most two points of the horizon. In simpler terms, this means that when we combine two gauge fields, denoted as A^1_μ and A^2_μ, belonging to the Gribov region, their sum $\lambda A^1_\mu + \sigma A^2_\mu$ will also lie within this region if the coefficients λ and σ satisfy $\lambda + \sigma = 1$ and $\lambda, \sigma \geq 0$. This result has been established and demonstrated in [46] and [47].

Such restriction will be implemented by adding into the action (1.65) the Horizon function [47; 61; 62; 63],

$$H^{MAG}(A) = g^2 \int d^4x d^4y f^{ab3} A^3_\mu(x) [M^{-1}]^{ac}(x,y) f^{cb3} A^3_\mu(y), \tag{2.50}$$

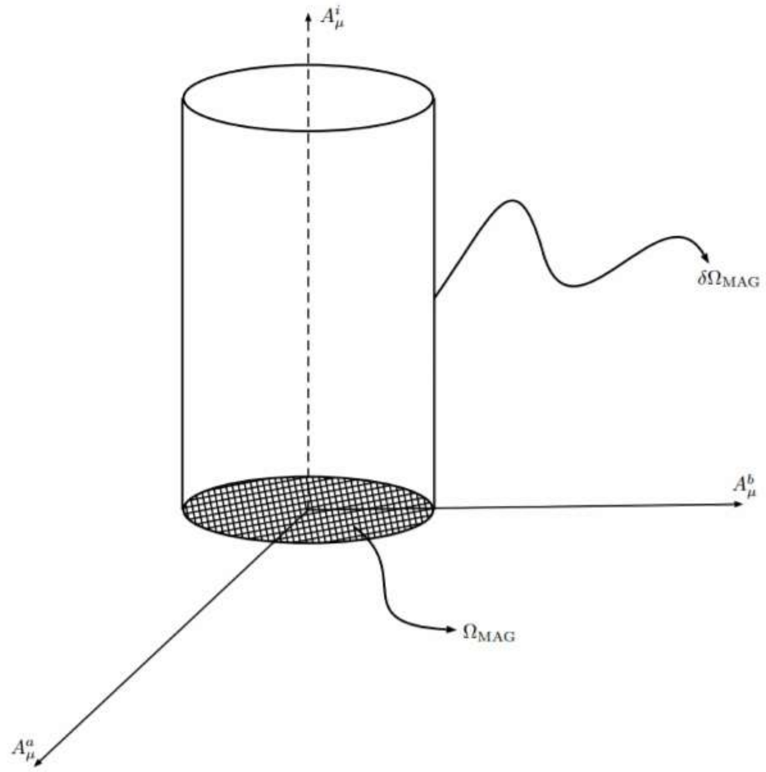


Figure 2.2: Gribov region in the MAG [3]

in such a way that,

$$S_{GZ}^{MAG} = S_{YM} + S_{FP}^{MAG} + \gamma^4 H^{MAG}(A) + 4\gamma^4 V(N^2-1), \tag{2.51}$$

with S_{FP}^{MAG} being given by eq. (1.66). As in Linear covariant gauges, the Horizon function (2.50) is non-local. It can be localized by means of suitable auxiliary fields, as can be seen on [44; 64; 65]. Doing so, we obtain the following local and renormalizable [64] action,

$$S_{GZ}^{MAG} = S_{YM} + S_{FP}^{MAG} + S_{\phi\omega}^{MAG} + S_{\gamma}^{MAG}, \tag{2.52}$$

with,

$$S_{\phi\omega}^{MAG} = \int d^4x d^4y (\bar{\phi}_\mu^{ac}(x) M^{ab}(x,y) \phi_\mu^{bc}(y) - \bar{\omega}_\mu^{ac}(x) M^{ab}(x,y) \omega_\mu^{bc}(y)), \quad (2.53)$$

and,

$$S_\gamma^{MAG} = \frac{g\gamma^2}{2} \int d^4x f^{ab3} A_\mu^3(x)(\phi_\mu^{ab}(x) + \bar{\phi}_\mu^{ab}(x)), \quad (2.54)$$

while the first two terms of eq.(2.52) are respectively given by equations (1.66) and (1.63).

From the action (2.51) we obtain the propagators,

$$\langle A_\mu^a(k) A_\mu^b(-k) \rangle = \frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}, \quad (2.55)$$

$$\langle A_\mu^3(k) A_\mu^3(-k) \rangle = \frac{k^2}{(k^4 + 2Ng^2\gamma^4)}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\delta^{33}, \quad (2.56)$$

where γ is the Gribov parameter. It can be seen from the expressions above that the elimination of the infinitesimal Gribov ambiguities only affects the Abelian sector of the YM theory, leading it to become supressed in the non-perturbative (infrared) regime. This happens because the Gribov parameter leads the poles of the Abelian propagator to acquire complex values, meaning that the Abelian gluon masses get out from the physical spectrum in the non-perturbative regime. This can be interpreted as an evidence of gluon confinement, since if a particle is confined, we expect its mass to be unphysical, since we can not measure it.

Regarding the BRST symmetry of the action , in the MAG they are given by,

$$
\begin{aligned}
sA^a_\mu &= -(\mathcal{D}^{ab}_\mu c^b + gf^{ab3}A^b_\mu c^3)\,, & sc^a &= gf^{ab3}c^b c^3\,, \\
sA^3_\mu &= -(\partial_\mu c^3 - gf^{ab3}A^a_\mu c^b)\,, & sc^3 &= 0\,, \\
s\bar{c}^a &= ib^a\,, & sb^a &= 0\,, \\
s\bar{c}^3 &= ib^3\,, & sb^3 &= 0\,, \\
s\phi^{ab}_\mu &= \omega^{ab}_\mu\,, & s\omega^{ab}_\mu &= 0\,, \\
s\bar{\omega}^{ab}_\mu &= \bar{\phi}^{ab}_\mu\,, & s\bar{\phi}^{ab}_\mu &= 0\,,
\end{aligned} \tag{2.57}
$$

where s is the nilpotent BRST operator. As well as in the Landau gauge, we have that the auxiliary field ω^{ab}_μ can be transformed as,

$$
\omega^{ab}_\mu \rightarrow \omega^{ab}_\mu + \frac{\gamma^2}{2}\int d^4y[M^{-1}]^{ac} f^{cb3}(\partial_\mu c^3 + gf^{de3}A^d_\mu c^e), \tag{2.58}
$$

and applying it on the action (2.52) we find that,

$$
\begin{aligned}
S^{MAG}_{GZ} &= S_{YM} + S^{MAG}_{FP} + S^{MAG}_{\phi\omega} + \gamma^4 H^{MAG}(A) + 4\gamma^4 V(N^2-1) + \\
&+ \frac{\gamma^2}{2g}\int d^4x(\mathcal{D}^{ab}_\mu \phi^{ba}_\mu - s(\mathcal{D}^{ab}_\mu \bar{\omega}^{ba}_\mu)),
\end{aligned} \tag{2.59}
$$

Now applying the BRST operator s on the action above we obtain,

$$
\begin{aligned}
sS^{MAG}_{GZ} &= s\int d^4x(\frac{\gamma^2}{2g}\mathcal{D}^{ab}_\mu \phi^{ba}_\mu) \\
&= -\frac{\gamma^2}{2}\int d^4x[f^{ab3}A^3_\mu \omega^{ba}_\mu - f^{ab3}(\partial_\mu c^3 + gf^{cd3}A^c_\mu c^d)\phi^{ba}_\mu] \\
&\neq 0,
\end{aligned} \tag{2.60}
$$

meaning that the BRST symmetry is softly broken as in the Landau gauge, since due to the presence of the γ^2 term, we have that the breaking terms vanish for

$\gamma \to 0$.

2.4 Refined Gribov-Zwanziger in the linear covariant gauges

As it was shown in the previous sections, the localization of the Gribov-Zwanziger action is implemented by means of the introduction of pairs of bosonic and fermionic auxiliary fields. In the linear covariant gauges such a procedure leads to the resulting local action (2.32). From that we realize the existence of a scaling behaviour for the form factor of the gluon propagator that vanishes in the limit $k \to 0$. However, lattice simulations [66; 67; 68; 69] have already showed that the zero-momentum limit of the form factor of the gluon propagator has actually a finite value. This means that the Gribov-Zwanziger action was was not compatible with such gluon propagator behaviour. There could yet exist some other features that were not being taken into account. Fortunately, it was observed in [14; 70] that non-trivial condensates are formed due to infrared instabilities of the GZ action [15].

With this in mind, it was also shown in [14] that we can take such condensates into account from the beginning by introducing into the action a term of the kind,

$$S_O = \int d^4x \left(JO - \rho g^2 J\right) , \tag{2.61}$$

where O is a 2-dimensional operator, J is the source, and ρ is a dimensionless parameter needed in order to grant the renormalizability of the theory since it is responsible for removing any potential divergencies proportional to $g^2 J$. Moreover we have that J and ρ form a BRST doublet, since,

$$s\rho = J\,, \qquad\qquad sJ = 0\,. \tag{2.62}$$

As the source J has a mass squared dimension we can rewrite $J = M^2$, where M^2 will be the mass of condensate $\langle O\rangle$ and will enter into the poles of the propagator modifying its physical spectrum. The resulting action of such a procedure is called as the Refined Gribov-Zwanziger action (RGZ). In the linear covariant gauges the RGZ action is given by [71],

$$S_{RGZ} = S_{GZ} + S_m + S_\mu, \tag{2.63}$$

where,

$$S_m = \int d^4x \left(\frac{1}{2} m^2 A^{h,a}_\mu A^{h,a}_\mu \right), \tag{2.64}$$

$$S_\mu = \int d^4x \left(-\mu^2 (\bar{\phi}^{ab}_\mu \phi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu) \right). \tag{2.65}$$

are respectively the actions that give the contribution of the condensation of the 2-dimensional operators $A^{h,a}_\mu A^{h,a}_\mu$ and $\bar{\phi}^{ab}_\mu \phi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu$. From eq. (2.63) we obtain that the gluon propagator in the linear covariant gauges becomes,

$$\langle A^a_\mu(k) A^b_\nu(-k)\rangle = \left[\frac{k^2+\mu^2}{(k^2+m^2)(k^2+\mu^2)+2Ng^2\gamma^4} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \frac{\alpha k_\mu k_\nu}{k^4} \right] \delta^{ab}. \tag{2.66}$$

From this propagator one can see that in the limit of $k \to 0$, and taking $\alpha \to 0$, we obtain,

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle\Big|_{k\to 0} = \frac{\mu^2}{m^2\mu^2+2Ng^2\gamma^4}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}. \tag{2.67}$$

This shows that at low momenta the propagator reach a finite value, which is in good agreement with the lattice results [66; 67; 68; 69], even at $1Gev$ and in [69].

It is important to state that the mass parameters m^2 and μ^2 are determined each one by their own gap equation which are obtained by following the procedures discussed in [71]. Yet, regarding the Gribov parameter γ^4, we have that in the RGZ context it will be determined by the following gap equation,

$$1 = \frac{3}{4}Ng^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2+m^2)(q^2+\mu^2)+2Ng^2\gamma^4}, \tag{2.68}$$

which was derived following the same steps that we took to derive eq. (2.13).

2.5 Refined Gribov-Zwanziger in the maximal Abelian gauge

As in the Linear covariant gauges, we can also analyse the condensation of two dimensional operators in the infrared regime of the Yang-Mills theory in the Maximal Abelian gauge.

To do so, we will start by adding to the action (2.52) the operators $A^a_\mu A^a_\nu$, $\bar{\phi}^{ab}_\mu\phi^{ab}_\mu - \bar{\omega}^{ab}_\mu\omega^{ab}_\mu$ by means of the same procedure shown on eq. (2.61). Here the operator $A^a_\mu A^a_\nu$ does it is not written in terms of the non-local gauge field A^h_μ, however this implies that the action will not be BRST invariant, as in the linear covariant case. Therefore, the refined Gribov-Zwanziger action in such gauge becomes,

$$S_{RGZ}^{MAG} = S_{GZ}^{MAG} + \int d^4x \left(\frac{M^2}{2} A_\mu^a A_\mu^a - \mu^2 (\bar{\phi}_\mu^{ab} \phi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}) \right), \quad (2.69)$$

where the mass μ stands for the condensate $\langle \bar{\phi}_\mu^{ab} \phi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab} \rangle$ while M, for $\langle A_\mu^a A_\mu^a \rangle$. Then, analogously to [50; 72; 73; 74], we obtain that the non-Abelian and Abelian gluon propagators are, respectively,

$$\langle A_\mu^a(k) A_\nu^b(-k) \rangle = \frac{1}{(k^2 + M^2)} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \delta^{ab}, \quad (2.70)$$

$$\langle A_\mu^3(k) A_\nu^3(-k) \rangle = \frac{(k^2 + \mu^2)}{[k^2(k^2 + \mu^2) + 2Ng^2\gamma^4]} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \delta^{33}. \quad (2.71)$$

One can see here that the mass parameter M that comes form the condensation of the two dimensional operator $\langle A_\mu^a A_\mu^a \rangle$ affects the non-Abelian gluon sector of the theory, while the mass parameter μ that comes form the condensation of the operator $\langle \bar{\phi}_\mu^{ab} \phi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab} \rangle$ affects the Abelian one.

Taking the limit $k \to 0$ we find that,

$$\langle A_\mu^a(k) A_\nu^b(-k) \rangle \mid_{k \to 0} = \frac{1}{M^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \delta^{ab}, \quad (2.72)$$

$$\langle A_\mu^3(k) A_\nu^3(-k) \rangle \mid_{k \to 0} = \frac{\mu^2}{\gamma^4} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \delta^{33}, \quad (2.73)$$

which means that at low momenta the form factors of the Abelian and non-Abelian sectors will reach a finite value, differently from what happens without the Gribov parameter and the masses of the condensates. It is important to say that such masses were already fitted with lattice on [75] and the results agreed with the Abelian dominance conjecture, since the non-Abelian mass M shows to be much bigger than μ. In other words this means that the non-Abelian propagator has a shorter range than the Abelian one.

Another important result we can obtain from (2.71) repeating the same steps followed in section 2.4 is the new gap equation,

$$\frac{g^2}{2}\int\frac{d^4k}{(2\pi)^3}\frac{1}{k^2(k^2+\mu^2)+2Ng^2\gamma^4}=1, \tag{2.74}$$

from which becomes evident that the condensation of the one dimensional operators will affect the energy of the lowest energy state, since the gap equation gives the difference between the vacuum energy state and the energy of the first excited state.

Chapter 3

The Yang-Mills-Chern-Simons theory

In this chapter, we will examine the Yang-Mills-Chern-Simons theory within the Gribov-Zwanziger scenario. Our focus will be on how the addition of the Chern-Simons term and the elimination of infinitesimal Gribov copies influence the propagators of the 3-dimensional Euclidean Yang-Mills theory. We will perform our calculations in both the Linear covariant gauges with $SU(N)$ symmetry and in the Maximal Abelian gauges for the $SU(2)$ case.

One of the relevant aspects of 3-dimensional Euclidean Yang-Mills-Chern-Simons theories lies in their connection to high-temperature Quantum Chromodynamics (QCD) [9]. At high temperatures, the quarks decouples from the gluons, forming a plasma of free quarks, and the gluonic sector of QCD becomes a pure Yang-Mills theory. The high-energy regime of the 4-dimensional Minkowski Yang-Mills theory can then be described by an effective Yang-Mills-Higgs model [8; 9; 76; 77; 78] in the 3-dimensional Euclidean space [9]. The Higgs term $(D_i A_0)^2$ introduces a mass to the gauge field propagators by breaking the symmetry of their vacuum expectation values. In this regime, 4-dimensional integrals over

4-momentum become 3-dimensional integrals over Euclidean 3-momentum.

As noted in [8], the Yang-Mills-Chern-Simons and Yang-Mills-Higgs theories are analogous from a qualitative point of view, as they both have the same number of massive physical degrees of freedom. However, the mechanism for generating mass is different in these two theories. In the Yang-Mills-Chern-Simons model, the mass is generated by the topological Chern-Simons term.

The Yang-Mills-Chern-Simons (YMCS) action in three Euclidean dimensions with $SU(N)$ gauge group is expressed as,

$$S_{\text{YMCS}} = S_{\text{YM}} + S_{\text{CS}}\,, \tag{3.1}$$

where the Chern-Simons action S_{CS} being

$$S_{\text{CS}} = -iC \int d^3x \epsilon_{\mu\rho\nu} \left(\frac{1}{2} A^a_\mu \partial_\rho A^a_\nu + \frac{g}{3!} f^{abc} A^a_\mu A^b_\rho A^c_\nu \right)\,, \tag{3.2}$$

with C being a mass parameter and $\epsilon_{\mu\rho\nu}$ is the totally anti-symmetric Levi-Civita symbol. As we can see the metric does not appear explicitly in the Chern-Simons action, since we do not have a scalar product between vectors. This implies that C is a mass of topological nature [8].

One important feature of the action (3.2) is that it is invariant under infinitesimal gauge transformations, as we can see in [8; 79; 80]. However, under finite gauge transformations the Chern-Simons Lagrangian L_{CS} transform as,

$$L_{CS} \to L_{CS} + \frac{iC}{4} \epsilon_{\mu\nu\alpha} \partial_\mu tr(\partial_\nu u u^{-1} A_\alpha) + \frac{12 i \pi^2 C}{3!} n(u), \tag{3.3}$$

where the second term is a total derivative, while,

$$n(u) = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha} tr(u^{-1}\partial_\mu u u^{-1} \partial_\nu u u^{-1} \partial_\alpha u), \tag{3.4}$$

is the so called winding number density of the group element u, whose integral must be an integer in order to make the Chern-Simons Boltzmann factor to be gauge invariant under finite gauge transformations.

3.1 Quantization of the $SU(N)$ Yang-Mills-Chern-Simons theory in Linear covariant gauges

Following the prescription of section 2.4 and the work of Fabrizio Canfora [13; 81] we have that the YMCS partition function quantized in the linear covariant gauges and restricted to the region free of many infinitesimal Gribov copies will be,

$$\mathcal{Z} = \mathcal{N} \int [D\bar{\phi}][D\phi][D\bar{\omega}][D\omega][Dc][D\bar{c}][Db][D\tau][D\bar{\eta}][D\eta][DA]e^{-S_{GZ}-3V\gamma^4(N^2-1)} \tag{3.5}$$

where $S_{\rm GZ}$ is the BRST invariant form of the YMCS action free from infinitesimal Gribov copies, being given by,

$$\begin{aligned} S_{\rm GZ} &= S^{\rm LCG}_{\rm YMCS} - \int d^3x \Big(\bar{\phi}^{ac}_\mu \mathcal{M}^{ab}(A^h)\phi^{bc}_\mu - \bar{\omega}^{ac}_\mu \mathcal{M}^{ab}(A^h)\omega^{bc}_\mu\Big) + \\ &+ \gamma^2 \int d^3x\, g f^{abc} A^{h,a}_\mu (\bar{\phi}+\phi)^{bc}_\mu + \int d^3x \Big(\tau^a \partial_\mu A^{h,a}_\mu - \bar{\eta}^a \mathcal{M}^{ab}(A^h)\eta^b\Big) , \end{aligned} \tag{3.6}$$

with,

$$S^{\rm LCG}_{\rm YMCS} = S_{\rm YMCS} + \int d^3x \left(b^a \partial_\mu A^a_\mu - \frac{\alpha}{2} b^a b^a + \bar{c}^a \partial_\mu D^{ab}_\mu c^b\right) . \tag{3.7}$$

The tree-level gauge-field propagator arising from the action (3.6) will be,

$$\begin{aligned}\langle A^a_\mu(p)A^b_\nu(-p)\rangle &= \delta^{ab}\left[\frac{p^2(p^4+2g^2\gamma^4N)}{(p^4+2g^2\gamma^4N)^2+C^2p^6}\left(\mathcal{P}^{\mathrm{T}}_{\mu\nu}(p)+\frac{Cp^2\left(\epsilon_{\mu\lambda\nu}p_\lambda\right)}{p^4+2g^2\gamma^4N}\right)+\right.\\ &+ \left.\alpha\frac{p_\mu p_\nu}{p^4}\right],\end{aligned} \tag{3.8}$$

with,

$$\mathcal{P}^{\mathrm{T}}_{\mu\nu}(p)=\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}, \tag{3.9}$$

being the transverse projector. If we take $\gamma\to 0$ or $p\to\infty$ we can see that,

$$\langle A^a_\mu(p)A^b_\nu(-p)\rangle = \delta^{ab}\left[\frac{1}{p^2+C^2}\left(\mathcal{P}^{\mathrm{T}}_{\mu\nu}(p)+\frac{C}{p^2}\epsilon_{\mu\lambda\nu}p_\lambda\right)+\alpha\frac{p_\mu p_\nu}{p^4}\right], \tag{3.10}$$

we obtain the gauge propagator of the ultraviolet regime of the theory, which means the propagator of the perturbative regime [71; 81], that is the regime in which the coupling constant g is $g << 1$. Thus, we obtain in such limit the regime in which the infinitesimal Gribov ambiguities are negligible.

From eq. (3.8) we can observe that there are two different sectors that can be analysed; the parity-preserving and the parity-violating ones. The parity-violating sector is the one multiplied by $\epsilon_{\mu\lambda\nu}$, while the parity preserving is all the rest.

The parity-violating sector, as we can see, is deeply affected by the eliminations of the infinitesimal Gribov ambiguities but it is not dependent on the gauge parameter α. Yet regarding the parity-preserving sector, we can see that only the transverse components were affected by the Gribov massive parameter γ and the Chern-Simons mass C. We can also see that if we take $C \to 0$ we obtain,

$$\langle A^a_\mu(p) A^b_\nu(-p)\rangle \;=\; \delta^{ab}\left(\frac{1}{p^2}\mathcal{P}^{\mathrm{T}}_{\mu\nu}(p) + \alpha\frac{p_\mu p_\nu}{p^4}\right).$$

which is the usual 3-dimensional gluon propagator in the linear covariant gauges. Moreover, the parity-preserving and transverse part of (3.8) does not depend on α at the tree level and coincides with the result in [13].

As we already saw in the previous section, the Gribov parameter is not a free parameter but determined by the so called gap equation, which in this case will be given at 1-loop by,

$$\frac{2Ng^2}{3}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^4+2g^2\gamma^4N}{(p^4+2g^2\gamma^4N)^2+C^2p^6} = 1\,. \tag{3.11}$$

This equation sets γ as a function of C and g. The 3-dimensional integral (3.11) is convergent and can be solved directly, but the result is a complicated expression with two free parameters, g and C. In contrast to theories without the Chern-Simons term, the massive parameter C brings a distinct mass scale to the theory. The solution for the gap equation in the presence of C can be found in [82].Additionally, the gap equation at one loop does not depend on the gauge parameter α. This is expected since the non-local form of the gap equation,

$$\langle H(A^h)\rangle = 3V(N^2-1) \tag{3.12}$$

is manifestly gauge invariant and does not depend on α. Since γ plays a role in gauge-invariant correlation functions, it has a physical character and should not depend on α.

Another important feature that can be observed form eq. (3.11) is that, due to the no pole condition,

$$\rho(0, A) < 1. \tag{3.13}$$

it follows that,

$$\frac{g^2}{6\pi C} < 1, \tag{3.14}$$

meaning that in the weak coupling regime the theory is automatically within the Gribov region.

More than that, the fact that the Chern-Simons term is not invariant under large gauge transformations, even though the partition function is gauge-invariant, has been noted in literature [8; 13]. To ensure the gauge-invariance of the partition function, we must impose a quantization rule for the Chern-Simons mass C given by $4\pi\frac{C}{g^2} = n$, where n is an integer $(1, 2, 3$, etc.). Substituting this into eq.(3.14), we find that for values of $n > \frac{2}{3}$, the no-pole condition (3.14) will always be satisfied, regardless of the value of C, implying that the Gribov copies associated with finite gauge transformations will not affect the results.

In other words, we can say that since the YMCS theories are invariant only under infinitesimal gauge transformations then just the infinitesimal copies are relevant and restricting the path integral to the Gribov region removes the infinitesimal copies.

3.1.1 Analytic structure of the gluon propagator

The propagator described in equation (3.8) is influenced by both the coupling constant g and the Chern-Simons mass C. Its pole structure is intricate and the poles are a function of these parameters. Understanding the behavior of the poles as g and C change is crucial in comprehending the various regimes in which the theory can exist, as discussed in recent studies of Yang-Mills theories with Higgs fields and gauge theories at finite temperature [8; 9; 13; 76; 77; 78]. In the plane of g and C, the region where the gauge propagator poles are complex, or negative, is considered to caracterize a confining regime, as both complex and negative poles cannot be linked to a physical excitation in the mass spectrum. Conversely, the region where the poles are real and have positive residues is seen as a deconfined area with a massive gauge particle in the spectrum.

This is so due to the fact that when a propagator has a pole with a positive residue, it means that it corresponds to a stable particle [83]. More than that, the positive residue indicates that the particle has a positive probability of existing and propagating through spacetime. These stable particles typically have real masses and are observed as long-lived entities.

On the other hand, when a propagator has a pole with a negative residue, it is usually interpreted as an unstable (short-lived) particle or a resonance. The negative residue indicates that the particle has a negative probability of existing [83], meaning that they are unphysical states. Thus, as we are talking about the non-perturbative regime of gauge fields, we will interpret such unphysical states as confined particles, since confined particles are not physical observables.

To determine the poles of the propagator in equation (3.8), we must locate the roots of the following polynomial,

$$\begin{aligned} P(p^2) &= (p^4 + 2g^2\gamma^4 N)^2 + C^2 p^6 \\ &= q^8 + C^2 q^6 + 2Gq^4 + G^2 \\ &= (p^2 + m_1)(p^2 + m_2)(p^2 + m_3)(p^2 + m_4), \end{aligned} \tag{3.15}$$

where we used $G = 2g^2\gamma^4 N$ in order to be able to compare our results with the ones of Canfora in [13].

To clarify our analysis of the pole structure of the gluon propagator, we will separate it in its two sectors; the parity preserving and the parity-violating ones, as follows,

$$\langle A_\mu(p)A_\nu(-p)\rangle = G_{\mu\nu}(p)\mid_{par} + G_{\mu\nu}(p)\mid_{par-viol}, \tag{3.16}$$

where,

$$G_{\mu\nu}(p)\mid_{par} = \frac{p^2(p^4+2g^2\gamma^4N)}{(p^4+2g^2\gamma^4N)^2+C^2p^6}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right), \tag{3.17}$$

$$G_{\mu\nu}(p)\mid_{par-viol} = \frac{Cp^4}{(p^4+2g^2\gamma^4N)^2+C^2p^6}\varepsilon_{\mu\sigma\nu}p_\sigma. \tag{3.18}$$

Now we make use of the partial fraction decomposition on equations (3.17) and (3.18) we obtain,

$$G_{\mu\nu}(p)\mid_{par} = \left(\frac{N_1}{p^2+m_1^2}+\frac{N_2}{p^2+m_2^2}+\frac{N_3}{p^2+m_3^2}+\frac{N_4}{p^2+m_4^2}\right)\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right), \tag{3.19}$$

$$G_{\mu\nu}(p)\mid_{par-viol} = \left(\frac{H_1}{p^2+m_1^2}+\frac{H_2}{p^2+m_2^2}+\frac{H_3}{p^2+m_3^2}+\frac{H_4}{p^2+m_4^2}\right)C\varepsilon_{\mu\sigma\nu}p_\sigma, \tag{3.20}$$

where N_1, N_2, N_3, N_4 and H_1, H_2, H_3, H_4 are, respectively, the residues of the parity-symmetric and parity violating parts of the propagator and are given by,

$$N_1 = \frac{m_1^2(\gamma^4+m_1^4)}{(m_1^2-m_2^2)(m_1^2-m_3^2)(m_1^2-m_4^2)}, \tag{3.21}$$

$$N_2 = \frac{-m_2^2(\gamma^4+m_2^4)}{(m_1^2-m_2^2)(m_2^2-m_3^2)(m_2^2-m_4^2)}, \tag{3.22}$$

$$N_3 = \frac{m_3^2(\gamma^4+m_3^4)}{(m_1^2-m_3^2)(m_2^2-m_3^2)(m_3^2-m_4^2)}, \tag{3.23}$$

$$N_4 = \frac{-m_4^2(\gamma^4+m_4^4)}{(m_1^2-m_4^2)(m_2^2-m_4^2)(m_3^2-m_4^2)}, \tag{3.24}$$

and,

$$H_1 = \frac{m_1^4}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)(m_1^2 - m_4^2)}, \tag{3.25}$$

$$H_2 = \frac{-m_2^4}{(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_2^2 - m_4^2)}, \tag{3.26}$$

$$H_3 = \frac{m_3^4}{(m_1^2 - m_3^2)(m_2^2 - m_3^2)(m_3^2 - m_4^2)}, \tag{3.27}$$

$$H_4 = \frac{-m_4^4}{(m_1^2 - m_4^2)(m_2^2 - m_4^2)(m_3^2 - m_4^2)}. \tag{3.28}$$

Such residues are important for the analysis of the degrees of freedom of the YMCS theory because in order to a pole be considered as a physical (deconfined) degree of freedom, it not only needs to be positive valued but also needs to have a correspondent positive valued residue [13; 81; 83]. Then in order to analyse the positivity or not of a residue, we need to analyse the discriminant of eq. (3.15), which is given by,

$$\Delta = 254C^4G^5 - 27C^8G^4 \tag{3.29}$$

We can distinguish two regimes in the theory, one with four complex masses ($\Delta > 0$ or $G > (3C/4)^4$) and another with two complex and two real masses ($\Delta < 0$ or $G < (3C/4)^4$). At low values of C and high values of G, all excitations are confined, while at high C and low G, real poles appear in the propagator and the theory becomes deconfined. Within the deconfined regime, there are two parts: one with intermediate values of C and G, where there are two massive poles, and another in the weak-coupling regime with large C, where the Gribov problem is not present and the theory resembles perturbative behavior.

By using computer algebra, the four roots of eq. (3.15) can be computed and plotted in terms of the theory's parameters. If the reader wants to see the plots of the real and complex parts of the masses m_1 and m_2 it is encouraged to see [13]. What can be seen there is that the masses for the parity-violating and conserved parts of the propagator are the same, but their residues differ slightly. In both cases, the residues of m_1 are positive, while those of m_2 are negative, meaning the state associated with m_2 cannot be physical.

This results are in qualitative agreement with those obtained in [84; 85] for Yang-Mills-Higgs theory. A regime was discovered with two masses with the corresponding residues being one positive and one negative. When the weak coupling condition is met, the standard particle spectrum, as described by gauge propagator (3.8), is obtained. It was also revealed in [81] that the Gribov gap equation in pure Yang–Mills theory at finite temperatures produced a phase diagram similar to those in [84; 85], with temperature serving as a substitute for the Higgs vacuum expectation value. This will also be seen in the next chapter were we will talk about such finite-temperature effective models in more details.

3.2 Quantization of the $SU(2)$ Yang-Mills-Chern-Simons theory into the MAG

As previously seen, the maximal Abelian gauge (MAG) is a specific choice of gauge in which the gauge fixing condition is chosen in order to be able to study the Abelian and non-Abelian sectors of the gauge fields separately, allowing us to study the the phenomenon of the Abelian dominance of the theory in the non-perturbative regime. This is why we will now try to understand how Yang-Mills-Chern-Simons (YMCS) theory behaves in the maximal Abelian gauge, in particular when infinitesimal Gribov copies are taken into account.

The Yang-Mills-Chern-Simons action in three Euclidean dimensions with $SU(2)$

gauge group is expressed as,

$$
\begin{aligned}
S_{YMCS} &= \frac{1}{4}\int d^3x \left(F^A_{\mu\nu}F^A_{\mu\nu} - iC\epsilon_{\mu\alpha\nu}\left(F^A_{\mu\nu}A^A_\alpha + \frac{g}{3}f^{ABC}A^A_\mu A^B_\nu A^C_\alpha\right)\right) \\
&= \frac{1}{4}\int d^3x \Bigg[\left(F^a_{\mu\nu}F^a_{\mu\nu} + F^3_{\mu\nu}F^3_{\mu\nu} - iC\epsilon_{\mu\alpha\nu}F^a_{\mu\nu}A^a_\alpha - iC\epsilon_{\mu\alpha\nu}F^3_{\mu\nu}A^3_\alpha\right) + \\
&- iC\epsilon_{\mu\alpha\nu}\left(\frac{g}{3}f^{3bc}A^3_\mu A^b_\nu A^c_\alpha + \frac{g}{3}f^{a3c}A^a_\mu A^3_\nu A^c_\alpha + \frac{g}{3}f^{ab3}A^a_\mu A^b_\nu A^3_\alpha\right)\Bigg], \quad (3.30)
\end{aligned}
$$

where we have explicitly decomposed the Abelian and non-Abelian terms of the action, and where,

$$
\begin{aligned}
F^a_{\mu\nu} &= \mathcal{D}^{ab}_\mu A^b_\nu - \mathcal{D}^{ab}_\nu A^b_\mu, \quad (3.31) \\
F^3_{\mu\nu} &= \partial_\mu A^3_\nu - \partial_\nu A^3_\mu + gf^{ab3}A^a_\mu A^b_\nu. \quad (3.32)
\end{aligned}
$$

In order to implement the MAG condition in eq. (3.30), we have to introduce a BRST-exact[1] term given by eq. (1.66). Hence, the Faddeev-Popov quantization of the Yang-Mills-Chern-Simons in the MAG is given by the path integral

$$
\mathcal{Z}_{\text{MAG}} = \int [\mathcal{D}\mu_{\text{YMCS}}]\ \mathrm{e}^{-S}\,, \quad (3.33)
$$

with $S = S_{\text{YMCS}} + S^{\text{MAG}}_{\text{FP}}$ and $[\mathcal{D}\mu_{\text{YMCS}}] = [\mathcal{D}A]\,[\mathcal{D}b]\,[\mathcal{D}\bar{c}]\,[\mathcal{D}c]$.

The tree-level propagators for the non-Abelian and Abelian gauge fields are, respectively,

$$
\langle A^a_\mu(k)A^b_\nu(-k)\rangle = \frac{\delta^{ab}}{k^2+C^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + C\frac{k_\rho}{k^2}\epsilon_{\mu\rho\nu}\right)\,, \quad (3.34)
$$

and

$$
\langle A_\mu(k)A_\nu(-k)\rangle = \frac{1}{k^2+C^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + C\frac{k_\rho}{k^2}\epsilon_{\mu\rho\nu}\right)\,. \quad (3.35)
$$

[1]See the appendix of [74] for more details on the BRST transformations.

The tree-level result depicted in expressions (3.34) and (3.35) reveals that both the Abelian and non-Abelian sectors of the gauge field have obtained contribution of a topological mass. While their propagators are transverse, this may not remain accurate in the non-Abelian sector at higher orders due to the non-linear gauge condition specified in (1.66).

As it was shown in chapter 2 the MAG is faced with the existence of the Gribov copies, as demonstrated in various studies including [86]. By confining the functional integral to the Gribov region, the Gribov problem is partially resolved. In fact, the Gribov region is free from infinitesimal Gribov copies and, in this case, the presence of the Chern-Simons term ensures that gauge invariance is upheld only for infinitesimal transformations. Thus, for Yang-Mills-Chern-Simons theory, removing the infinitesimal Gribov copies actually constitutes a full solution to the Gribov problem, as seen on [74; 87]. However, to guarantee the invariance under finite gauge transformation, the mass parameter C must comply with a quantization rule, as stated in [74; 79; 80].

In order to eliminate the infinitesimal Gribov copies we have to restrict our path integral to the Gribov region, which in the maximal Abelian gauge will be, as we saw in the last chapter, given by,

$$\Omega_{\text{MAG}} = \left\{ (A^a_\mu, A^3_\mu), \mathcal{D}^{ab}_\mu A^b_\mu = 0 \,, \partial_\mu A^3_\mu = 0 \,|\, \mathcal{M}^{ab} > 0 \right\} \,. \tag{3.36}$$

Such a restriction can be achieved through the application of the Gribov no-pole condition. This entails ensuring that the ghost propagator has a single pole at $p^2 = 0$. This procedure was previously established in pure Yang-Mills theories (see [86]), and can be applied to the Yang-Mills-Chern-Simons theory in three dimensions by imposing the condition on the non-Abelian ghost fields. The restriction is a geometrical process that can be carried out for any dimension of spacetime.

At leading order in the coupling g, the ghost two-point function is

$$\begin{aligned} \mathcal{G}(p) &= \frac{1}{2V}\sum_{a=1,2}\langle \bar{c}^a(p)c^a(-p)\rangle_A \\ &= \frac{1}{p^2}(1+\sigma(p,A))+\frac{B}{p^4}\,, \end{aligned} \tag{3.37}$$

where V stands for the spacetime volume, $\langle \ldots \rangle_A$ denotes that the correlation function is computed by taking A as an external field and,

$$\sigma(p,A) = \frac{4g^2}{3V}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{A_\alpha(k)A_\alpha(-k)}{(p-k)^2}\,, \tag{3.38}$$

where B is independent of p and it is positive. Thus, only $\sigma(p,A)$ can cause a pole that isn't trivial with a value of $p^2=0$, and the terms featuring B are not relevant to the current discussion.

The generation of a non-trivial pole can be prevented if $\sigma(p,A)<1$. This condition is satisfied if $\sigma(p,A)$ decreases monotonically with p^2. To enforce the restriction that no non-trivial poles are generated, the condition $\sigma(0,A)<1$ can be imposed, referred to as the no-pole condition. Such a condition will be implemented in the partition function (3.33) by adding a step function $\theta(1-\sigma(0,A))$ as follows,

$$\mathcal{Z}_{\mathrm{MAG}} = \int [\mathcal{D}\mu_{\mathrm{YMCS}}]\ \mathrm{e}^{-S}\theta(1-\sigma(0,A))\,, \tag{3.39}$$

where,

$$\theta(1-\sigma(0,A)) = \int_{-i\infty+\epsilon}^{i\infty+\epsilon}\frac{\mathrm{d}\zeta}{2\pi i\zeta}\mathrm{e}^{\zeta(1-\sigma(0,A))}\,. \tag{3.40}$$

It's possible to translate the change to the path integral measure into a factor in the action, which results in a modified gluon propagator. This modification features $\sigma(0,A)$, which encompasses the Abelian gauge fields. Upon considering only quadratic terms over the fields in the partition function and integrating out all other fields, the following expression is obtained,

$$\mathcal{Z}_{\rm MAG}^{\rm quad} = \lim_{\alpha,\beta\to 0} \mathcal{N} \int \frac{{\rm d}\zeta}{2\pi i} {\rm e}^{\zeta - \ln\,\zeta} {\rm det}^{-1/2} \Delta_{\mu\nu}^{ab}\, {\rm det}^{-1/2} \tilde{\Delta}_{\alpha\beta},\,, \tag{3.41}$$

where $\mathcal{N}$ is a normalization constant and,

$$\Delta_{\mu\nu}^{ab} = \delta^{ab}\left[\delta_{\mu\nu}p^2 - \left(1+\frac{1}{\alpha}\right)p_\mu p_\nu - C\epsilon_{\mu\rho\nu}p_\rho\right]\,, \tag{3.42}$$

$$\tilde{\Delta}_{\mu\nu}^{33} = \delta^{33}\left[\delta_{\mu\nu}\left(p^2 + \frac{8g^2}{3V}\frac{\zeta}{p^2}\right) - \left(1+\frac{1}{\beta}\right)p_\mu p_\nu - C\epsilon_{\mu\rho\nu}p_\rho\right]. \tag{3.43}$$

where the α and β parameters are gauge parameters that enter in the effective action of the theory after integrating out the Lautrup-Nakanishi fields b^a and b^3, respectively, in the path integral (see [46]).

The determinant of the operator $\Delta_{\mu\nu}^{ab}$ will be absorbed in the normalization factor by defining $\mathcal{N}' \equiv \mathcal{N}\,{\rm det}^{-1/2}\Delta_{\mu\nu}^{ab}$ since it does not depend on ζ. Then, we will make use of the saddle-point method in order to evaluate the integral that we obtain after that, giving us,

$$\mathcal{Z}_{\rm MAG}^{\rm quad} = \mathcal{N}'\,{\rm e}^{f(\zeta^*)}\,, \tag{3.44}$$

with,

$$f(\zeta) = \zeta - \ln\,\zeta - \frac{1}{2}{\rm Tr}\ln\,\tilde{\Delta}_{\mu\nu}^{33}\,, \tag{3.45}$$

and ζ^* being the solution of,

$$\frac{\partial f(\zeta)}{\partial\zeta}\Big|_{\zeta=\zeta^*} = 0\,. \tag{3.46}$$

This leads to the expression,

$$1-\frac{1}{\zeta^*}-\frac{1}{2}\mathrm{Tr}\left[\frac{\partial\tilde{\Delta}^{33}_{\mu\alpha}}{\partial\zeta}\left(\tilde{\Delta}^{33}_{\alpha\nu}\right)^{-1}\right]\Bigg|_{\zeta=\zeta^*}=0\,, \tag{3.47}$$

where after taking the limits $V\to\infty$ and $\beta\to 0$ while holding $\gamma^4\equiv\frac{8g^2\zeta^*}{3V}$ finite yields

$$\frac{8g^2}{3}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^4+\gamma^4}{C^2p^6+(p^4+\gamma^4)^2}=1\,. \tag{3.48}$$

which is the so-called gap equation which determines the massive Gribov parameter γ in terms of the coupling constant g and the Chern-Simons mass C.

It is important to say that, as in the linear covariant case, the no-pole condition $\sigma(0,A)<1$ leads to,

$$\frac{g^2}{6\pi C}<1, \tag{3.49}$$

which together with the quantization rule $4\pi\frac{C}{g^2}=n$ will guarantee that the elimination of the infinitesimal Gribov copies is enough to grant the elimination of all the Gribov copies of the path integral of the YMCS theory.

We can finally compute the propagators of the theory from eq.(3.42) and (3.43), yielding,

$$\langle A^a_\mu(p)A^b_\nu(-p)\rangle = \frac{1}{p^2+C^2}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}+\frac{C}{p^2}\varepsilon_{\mu\sigma\nu}p_\sigma\right)\delta^{ab}, \tag{3.50}$$

$$\langle A^3_\mu(p)A^3_\nu(-p)\rangle = \frac{(p^4+\gamma^4)p^2}{(p^4+\gamma^4)^2+C^2p^6}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}+\frac{Cp^2}{(p^4+\gamma^4)}\varepsilon_{\mu\sigma\nu}p_\sigma\right)\delta^{33}, \tag{3.51}$$

where we can see that only the Abelian sector of the theory is affected by the restriction to the Gribov horizon since the Gribov parameter γ only appears in the poles of the Abelian gluon propagator. Furthermore, taking $C \to 0$ we reobtain the usual propagators of the pure 3-dimensional Yang-Mills theory. The effects of the Gribov parameter to the pole structure of the gluon propagator will be discussed in the next section.

Building upon the previous findings, we can now present the action associated with the Gribov-Zwanziger framework in the context of Yang-Mills-Chern-Simons theories that have been quantized in the MAG. The expression is as follows,

$$S_{GZ}^{MAG} = S_{YMCS} + S_{FP}^{MAG} + S_{\phi\omega}^{MAG} + S_{\gamma}^{MAG} + \gamma^4 H^{MAG}(A) + 4\gamma^4 V(N^2-1) \tag{3.52}$$

with[1] S_{FP}^{MAG}, $S_{\phi\omega}^{MAG}$, $H^{MAG}(A)$ and S_{γ}^{MAG} being respectively given by the equations (1.66), (2.53), (2.50) and (2.54). It has been demonstrated that when there is no topological mass present ($C = 0$), the action given in equation (3.52) is renormalizable in four dimensions according to [61]. However, the topic of the renormalizability of action (3.52) in the presence of the Chern-Simons is still an open question.

In this stage, various formal aspects related to action (3.52) can be examined. Of particular interest is the impact of the introduction of the horizon function on BRST symmetry, which leads to its soft breaking. This breaking is considered soft because in the extreme ultraviolet limit, the Gribov parameter vanishes, and BRST invariance is regained. The subject of BRST breaking has been widely explored in the literature [88; 89; 90; 91; 92; 93]. However, as mentioned in section (2.2.1) and on [55] for linear covariant gauges and extended to the MAG in [94], it is possible to have a manifestly BRST-invariant formulation of the horizon function. Despite these interesting aspects, we will instead concentrate

[1]We have performed a redefinition of the Gribov parameter $\gamma^2 \to g\gamma^2$ and Vol is a volume term whose explicit form is irrelevant for the present purposes.

on analyzing the analytic structure of the gluon propagator in the next section.

3.2.1 The analytic structure of the Abelian gluon propagator

To analyse the analytic structure of the Abelian gluon propagator basically means to analyse its pole structure in such a way to tell if it has physical degrees of freedom associated to them or not. In the present case, the poles of the propagator (3.51) are given by the roots of the following polynomial,

$$\begin{aligned} \mathcal{F}(p) &\equiv (p^4+\gamma^4)^2 + C^2p^6 = p^8 + \gamma^8 + 2p^4\gamma^4 + C^2p^6 \\ &= (p^2+m_1^2)(p^2+m_2^2)(p^2+m_3^2)(p^2+m_4^2)\,, \end{aligned} \tag{3.53}$$

with $(m_1^2, m_2^2, m_3^2, m_4^2)$ standing for the roots of (3.53). Decomposing the propagator (3.51) into parity-preserving and violating pieces as

$$\langle A_\mu(p)A_\nu(-p)\rangle = \mathcal{K}^{\text{pres}}_{\mu\nu}(p) + \mathcal{K}^{\text{viol}}_{\mu\nu}(p)\,, \tag{3.54}$$

with

$$\mathcal{K}^{\text{pres}}_{\mu\nu}(p) = \frac{p^2(p^4+\gamma^4)}{(p^4+\gamma^4)^2 + C^2p^6}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\,, \tag{3.55}$$

and

$$\mathcal{K}^{\text{viol}}_{\mu\nu}(p) = \frac{Cp^4}{(p^4+\gamma^4)^2 + C^2p^6}\epsilon_{\mu\lambda\nu}p_\lambda\,. \tag{3.56}$$

Each sector of the propagator is written in a partial-fraction like decomposition, leading to

$$\mathcal{K}^{\text{pres}}_{\mu\nu}(p) = \sum_{i=1}^{4}\frac{E_i}{p^2+m_i^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\,, \tag{3.57}$$

where

$$E_i = \frac{m_i^2(m_i^4+\gamma^4)}{\prod_{j=1\,,\,j\neq i}^{4}(m_i^2-m_j^2)}\,. \tag{3.58}$$

For the parity-violating sector, the decomposition is expressed as

$$\mathcal{K}^{\text{viol}}_{\mu\nu}(p) = \sum_{i=1}^{4}\frac{B_i}{p^2+m_i^2}\epsilon_{\mu\lambda\nu}p_\lambda\,, \tag{3.59}$$

with

$$B_i = -\frac{C^4 m_i^4}{\prod_{j=1\,,\,j\neq i}^{4}(m_i^2-m_j^2)}\,. \tag{3.60}$$

The pole and residue structure of the Abelian gluon propagator are evident in equations (3.57) and (3.59). The structure is dependent on the coupling constant g, the Gribov parameter γ, and the Chern-Simons mass C, which are linked by the gap equation (3.48). A full examination of the propagator's analytic structure would entail solving the gap equation to a certain degree of perturbation theory. However, this study, similar to [13], allows for the parameters to have unrestricted values and characterizes the theory's spectrum with those values. Though this approach gives a wider range of allowed parameter values, it does not rely on a fixed solution from perturbation theory and may allow for more extensive results in the future.

Now in order to be able to determine if the poles of the propagator are associated to confined or deconfined degrees of freedom, we need to analyse the roots of eq. (3.53), whose correspondent discriminant is,

$$\Delta = 256C^4\gamma^{20} - 27C^8\gamma^{16}\,. \tag{3.61}$$

In Fig. 3.1, the plot depicts the sign of the discriminant Δ as a function of γ^2 and C.

What we conclude by this is that if $\Delta > 0$, it is a known fact that either the

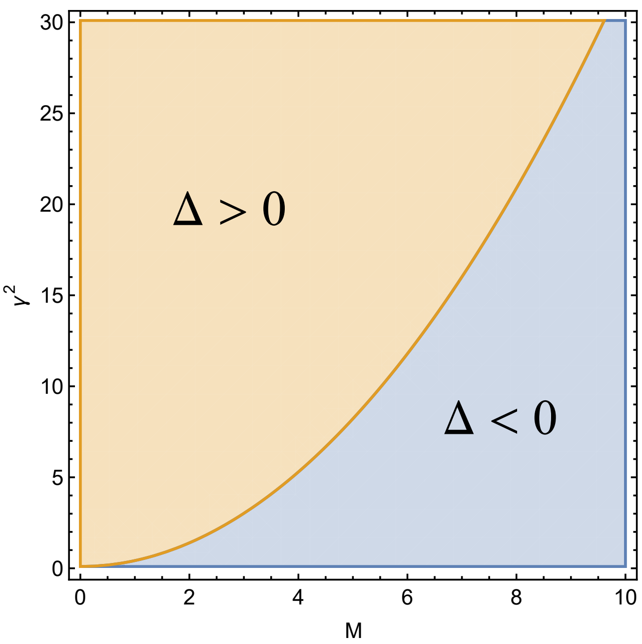


Figure 3.1: Sign of the discriminant Δ of $\mathcal{F}(p)$ as a function of C and γ^2.

poles are all real or all complex. This is established by the sign of subsidiary polynomials defined by,

$$\mathcal{P}(C,\gamma) = 16\gamma^4 - 3C^4\,, \tag{3.62}$$

and

$$\mathcal{D}(C,\gamma) = 32M^4\gamma^4 - 3C^8\,. \tag{3.63}$$

The presence of real roots in the system is dependent on two conditions, $\mathcal{P}(C,\gamma) < 0$ and $\mathcal{D}(C,\gamma) < 0$. However, when analyzing the parameter space with the chosen values of γ^2 and C, no overlap of regions fulfilling both conditions was found, thus resulting in no real roots. On the other hand, if either $\mathcal{P}(C,\gamma) > 0$ or $\mathcal{D}(C,\gamma) > 0$ with $\Delta > 0$, all roots are complex. The region where these conditions are simultaneously satisfied coincides with the area in Fig. 3.1 where $\Delta > 0$. This means that for a wide range of (γ^2, C) values, the poles are all complex and therefore, the excitations are not part of the physical spectrum.

This could be interpreted as evidence of confinement.

If Δ is negative, then $\mathcal{F}$ has two real roots and two conjugate complex roots. This can be seen in Fig.3.1 where the region where Δ is negative is indicated. Possibility of physical excitations being generated arises due to the two real roots. The signs of residues determine this. Parity-preserving and violating parts of the propagator have the same poles. For negative Δ, it is possible to find appropriate values of γ^2 and C so that the residue associated with one of the real poles is positive, which means it can be associated with a physical excitation. As a result, the theory can exhibit different regimes by changing the values of these "free" parameters - a confining regime, where all poles are complex and cannot correspond to physical excitations, and a different regime where physical excitations can appear. Fig.3.2 and 3.3 show the values of the residues for the parity-preserving and violating parts of the propagator.

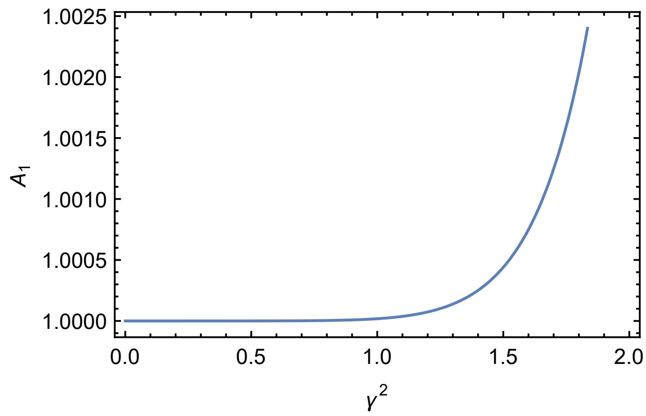


Figure 3.2: Residue A_1 - associated to the parity-preserving part of the propagator - for $C = -5$.

For the other real pole, the residues are negative and therefore, it cannot be associated to a physical excitation. The residues for $C = -5$, as a function of γ^2 are plotted in Fig. 3.4 and 3.5.

Thus, in the parameter space, there exists a region where the discriminant

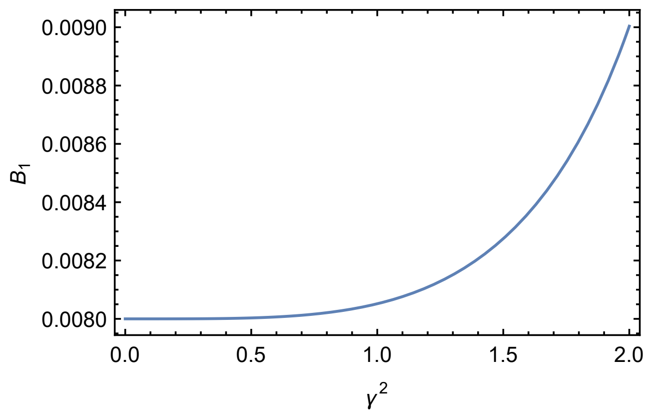


Figure 3.3: Residue B_1 - associated to the parity-violating part of the propagator - for $C = -5$.

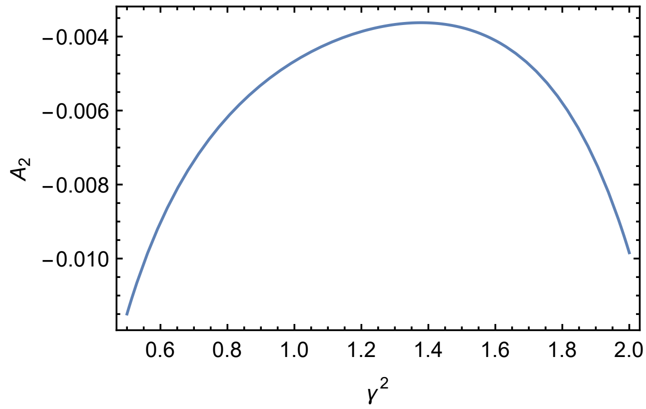


Figure 3.4: Residue A_2 - associated to the parity-preserving part of the propagator - for $C = -5$.

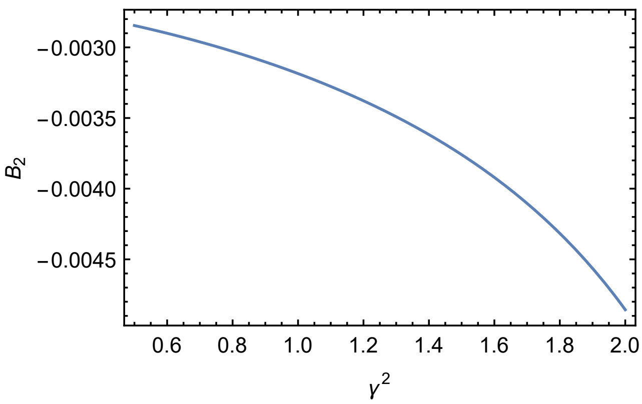


Figure 3.5: Residue B_2 - associated to the parity-violating part of the propagator - for $C = -5$.

Δ takes on a negative value and results in the creation of two real poles. One of these poles has positive residues, indicating the presence of a physical excitation. Conversely, the other real pole does not exhibit positive residues, as seen in Fig.3.4 and Fig.3.5. As a result, the theory showcases two distinct phases in the parameter space. In one phase, Δ is positive and all poles are complex, representing confined excitations. Meanwhile, in the other phase where Δ is negative, two real poles emerge, one of which with positive residues and is hence regarded as a deconfined excitation. This difference arises due to the interplay of the topological mass parameter C and the restriction to the Gribov region in γ^2. This transition from confined to deconfined phases has also been observed in Yang-Mills-Higgs and Yang-Mills-Chern-Simons-Higgs systems in the Landau gauge, as mentioned in references [76; 77; 84; 85] and [95]. However, this phenomenon only occurs in the Abelian sector and not in the non-Abelian sector due to the absence of competing mass parameters. Hence, the non-Abelian propagator exhibits a Yukawa-like behavior, and the Abelian-dominance hypothesis states that the mass should be large enough to separate these degrees of freedom in the infrared.

In 3 dimensions, the shift from confinement to deconfinement is a widely understood concept, first explored by Polyakov in his study of $SU(2)$ Yang-Mills theories linked to Higgs fields in the adjoint representation of the gauge group [96]. As seen in this study, there are two phases - one where all components of the gauge field are confined, and another where just one component, A^3_μ, is confined, with the phase transition being influenced by the gauge coupling g and the Higgs field's vacuum expectation value. However, the present model doesn't involve any additional fields and the phase transition solely occurs due to the existence of the topological mass. The restriction to the Gribov region leads to a Gribov-like propagator for the Abelian component A^3_μ of the gauge field, while the off-diagonal components always display a Yukawa-like behavior. This sets the present model apart from standard Yang-Mills theories linked to Higgs fields, as the off-diagonal components are always deconfined, and the Abelian component can have physical excitations based on the value of the topological mass. The criterion used in this work to determine confinement is based on the preservation of the propagator's reflection positivity, and the propagator was computed using the MAG condition, which is different from the gauge used by Polyakov, thus preventing a direct comparison of results. Despite this, the present model still captures the essence of phase transitions from confinement to deconfinement without the need for additional fields, and further exploration of this model in the presence of a Higgs field could provide deeper insights into the phase structure of 3-dim Yang-Mills theories.

Chapter 4

Finite temperatures

4.1 Introduction

In previous chapters, we explored the treatment of Yang-Mills theories, which involves introducing a gauge-fixing term to define the gauge field propagator. The Faddeev-Popov method is commonly employed in perturbation theory to fix the gauge. However, beyond perturbation theory, the assumptions of this procedure are no longer valid, leading to the existence of Gribov copies. Gribov and Zwanziger [36; 39; 44; 46; 49; 57] proposed addressing this issue by introducing the Gribov region, which is a region free of infinitesimal Gribov copies, in addition to the standard Faddeev-Popov gauge fixing. It is important to note that the choice of gauge should not impact the correlation functions of observables, and any proposed solutions to the Gribov problem in different gauges must take this into consideration. While the GZ and RGZ actions have shown promise in the Landau gauge [38; 46; 97], their applicability may not extend to other gauges. Therefore, addressing the Gribov problem in different gauges like the linear covariant gauges remains a complex task. Nevertheless, the study of the Gribov problem has provided valuable insights into the nature of gauge fixing and its implications for the underlying theory.

With all of this in mind, in this chapter we will analyse how the Yang-Mills theory within the GZ framework behaves at finite temperatures and we will identify how does the Gribov parameter affect the phase transitions of the theory and its critical temperatures. We will do so by means of a hybrid approach where the thermal masses are derived through perturbative propagators as a stepping stone for a self-consistent treatment. The resulting action collects the effects of the elimination of infinitesimal Gribov copies as well as the thermal masses. At the end we will verify the existence of three different phases for the gluonic degrees of freedom; one of complete confinement at low temperatures, an intermediate one of partial confinement, and one of complete deconfinement at high temperatures.

4.2 Immaginary time formalism

Finite-temperature Yang-Mills theory can be effectively investigated by employing the powerful mathematical framework known as the imaginary time formalism [81; 98; 99]. This formalism establishes a crucial connection between the generating functional of the corresponding quantum field theory and the partition function of a quantum statistical system. By compactifying the temporal coordinate, the imaginary time formalism allows for a rigorous exploration of thermal effects within the theory.

In this formalism, the length of the compactified time serves as a representation of the inverse temperature of a thermal bath. As such, it provides a comprehensive platform for analyzing the behavior of the system at finite temperatures. The partition function, which encapsulates the essential characteristics of the finite-temperature Yang-Mills theory, can be elegantly expressed within this framework by,

$$Z_{YM} = T\sum_{n=-\infty}^{\infty}\int [DA]exp\left(\frac{1}{4}\int_0^{1/T} d\tau \int d^3x F^A_{\mu\nu}F^A_{\mu\nu}\right), \tag{4.1}$$

where we made the transformation $t \to it$ over the temporal parameter t and we identified the temporal integration limits $[0, \frac{1}{T}]$, where T is the variable of temperature.

To facilitate calculations and analyses in momentum space, temperature-dependent fields $\phi(\tau, x)$ are expanded using a Fourier series over discrete Matsubara frequencies ω_n, in such a way that,

$$\begin{aligned} \phi(\tau, x) &= \int \frac{d^3k}{(2\pi)^3}(e^{-i\omega_n\tau - ik.x}\phi(\omega_n, k)), \\ \omega_n &= 2\pi n T. \end{aligned} \tag{4.2}$$

This choice arises from the identification of the temporal integration limits of $[0, T^{-1}]$ in the formalism. By considering these specific frequency components, we can investigate the intricacies of the YM theory in linear covariant gauges and extract valuable insights into its thermal properties and phenomena. In particular, in the next section we will calculate the gluon self-energy at finite temperatures

4.3 The gluon self-energy and the thermal mass

In this section we will calculate the thermal contribution to the poles of longitudinal and transverse sectors of the gluon propagator. This will be done by means of the components $\Pi^{(2)}_{44}$ and $\Pi^{(2)}_{\mu\mu}$ of the gluon self-energy $\Pi^{(2)}_{\mu\nu}$.

Then, to do so, we need first to write down the propagators and vertices of the theory in order to be able to build the diagrams that will contribute to the gluon self-energy.

From eq.(1.45) we obtain that in the linear covariant gauges the gluon and ghost propagators in the Euclidean space, given respectively by,

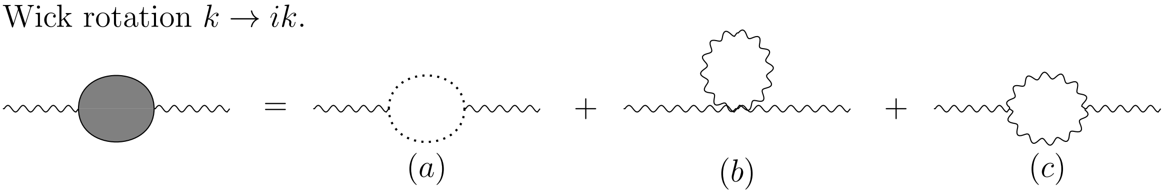
$$\begin{aligned}
\langle A^a_\mu(k) A^b_\nu(-k)\rangle &= \left[\frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \alpha \frac{k_\mu k_\nu}{k^4}\right]\delta^{ab}, \\
\langle \bar{c}^a(k) c^b(-k)\rangle &= -\frac{\delta^{ab}}{k^2}.
\end{aligned} \tag{4.3}$$

Regarding the vertices, we find that they are given by,

$$\begin{aligned}
V^{gfh}_{\alpha\beta\lambda}(q_1,q_2,q_3) &= (2\pi)^4 i g f^{gfh}\Bigg((q_1-q_3)_\beta\delta_{\alpha\lambda} + (q_3-q_2)_\alpha\delta_{\beta\lambda} + (q_2-q_1)_\lambda\delta_{\alpha\beta}\Bigg) \\
V^{gfhn}_{\alpha\beta\gamma\sigma}(q_1,q_2,q_3,q_4) &= -(2\pi)^4 g^2\Bigg(f^{abe}f^{cde}(\delta_{\alpha\gamma}\delta_{\sigma\beta} - \delta_{\sigma\alpha}\delta_{\beta\gamma}) + \\
&+ f^{ade}f^{bce}(\delta_{\alpha\beta}\delta_{\sigma\gamma} - \delta_{\alpha\gamma}\delta_{\sigma\beta}) + f^{ace}f^{bde}(\delta_{\alpha\beta}\delta_{\sigma\gamma} - \delta_{\sigma\alpha}\delta_{\beta\gamma})\Bigg) \\
V^{abc}_\mu(p,k,r) &= -(2\pi)^4 i g f^{abc} p_\mu
\end{aligned} \tag{4.4}$$

where p is the momentum of the $\bar{c}^b(p)$ field.

Now, we are able to write down the diagrams that contribute to the gluon self-energy. In order to do that, we will consider, for the construction of the vertices, a positive sign for all the momenta entering the vertex and a negative sign for the ones exiting from it. Moreover, as we will make calculations in the Minkowski space-time, we will write down the diagrams taking into account the Wick rotation $k \to ik$.



The diagram (a) is,

$$\begin{aligned}
i\Pi^{(2)ab}_{\mu\nu}(k) &= -g^2 f^{acd} f^{bc'd'} \int \frac{d^d p}{(2\pi)^d} \left(p_\mu \, (k+p)_\nu \left(-\frac{\delta^{cc'}}{p^2} \right) \left(-\frac{\delta^{dd'}}{(k+p)^2} \right) \right) \\
&= -g^2 f^{acd} f^{bcd} \int \frac{d^d p}{(2\pi)^d} \left(\frac{p_\mu \, (k+p)_\nu}{p^2 (k+p)^2} \right) \\
&= -g^2 N \delta^{ab} \int \frac{d^d p}{(2\pi)^d} \left(\frac{p_\mu \, (k+p)_\nu}{p^2 (k+p)^2} \right),
\end{aligned} \tag{4.5}$$

while the diagram (b) is,

$$\begin{aligned}
i\Pi^{(2)ab}_{\mu\nu}(k) &= \frac{g^2}{2} \Bigg(f^{abe} f^{cqe} \left(\delta_{\mu\delta}\delta_{\eta\nu} - \delta_{\eta\mu}\delta_{\nu\delta} \right) + f^{aqe} f^{bce} \left(\delta_{\mu\nu}\delta_{\eta\delta} - \delta_{\mu\delta}\delta_{\eta\nu} \right) + \\
&+ f^{ace} f^{bqe} \left(\delta_{\mu\nu}\delta_{\eta\delta} - \delta_{\eta\mu}\delta_{\nu\delta} \right) \Bigg) \int \frac{d^d p}{(2\pi)^d} \left[\frac{\delta^{cq}}{p^2} \left(\delta_{\delta\eta} - (1-\alpha) \frac{p_\delta p_\eta}{p^2} \right) \right] \\
&= g^2 N \delta^{ab} \int \frac{d^d p}{(2\pi)^d} \left(\frac{(1-\alpha) p_\mu p_\nu + (\alpha + d - 2)(p \cdot p) \delta_{\mu\nu}}{p^4} \right),
\end{aligned} \tag{4.6}$$

and the diagram (c) is,

$$\begin{aligned}
i\Pi^{(2)ab}_{\mu\nu}(k) &= \frac{1}{2} g^2 f^{acd} f^{c'bd'} \int \frac{d^d p}{(2\pi)^d} \Bigg[\Bigg((k-p)_\tau \delta_{\mu\lambda} + (p - (-k-p))_\mu \delta_{\tau\lambda} + \\
&+ (-k-p-k)_\lambda \delta_{\mu\tau} \Bigg) \Bigg((k+p-(-p))_\nu \delta_{\rho\delta} + (-p-(-k))_\rho \delta_{\nu\delta} + \\
&+ (-k-(k+p))_\delta \delta_{\rho\nu} \Bigg) \frac{\delta^{dd'}}{p^2} \left(\delta_{\lambda\delta} - (1-\alpha) \frac{p_\lambda p_\delta}{p^2} \right) \times \\
&\times \frac{\delta^{cc'}}{(k+p)^2} \left(\delta_{\tau\rho} - (1-\alpha) \frac{(k+p)_\tau (k+p)_\rho}{(k+p)^2} \right) \Bigg].
\end{aligned} \tag{4.7}$$

Making the transformations $t \to it$ and $p \to (p-k)$ and using the approximation $p >> k$ in the numerator, we find for the gluon-ghost diagram, eq.(4.5), that,

$$i\Pi^{(2)ab}_{\mu\nu}(k) = -g^2 N\delta^{ab} \int \frac{d^d p}{(2\pi)^d} \left(\frac{p_\mu p_\nu}{p^2(p-k)^2} \right) \tag{4.8}$$

Doing the same approximation on eq.(4.7), we find,

$$i\Pi^{(2)ab}_{\mu\nu}(k) = g^2 N\delta^{ab} \int \frac{d^d p}{(2\pi)^d} \left[\frac{\alpha}{p^4} \left(p_\mu p_\nu - p^2 \delta_{\mu\nu} \right) - \frac{(2(d-1)p_\mu p_\nu)}{p^4} \right]. \tag{4.9}$$

Thus, summing eqs.(4.6), (4.8) and (4.9) we obtain

$$i\Pi^{(2)ab}_{\mu\nu}(k) = 2g^2 N\delta^{ab} \int \frac{d^d p}{(2\pi)^d} \left(\frac{(p.p)\delta_{\mu\nu} - 2p_\mu p_\nu}{p^2(p-k)^2} \right). \tag{4.10}$$

As we can see, by using the High-Termal loop (HTL) approximation, we canceled the gauge parameter α dependence in the total self-energy.

Now we will calculate the components $\mu = \nu$ and $(\mu, \nu) = (4, 4)$ of the total self-energy (4.10) in order to calculate the thermal masses of the longitudinal and transversal sectors of the propagator of the Yang-Mills theory, given respectively by,

$$\Pi_L = \frac{K^2}{k^2} \Pi_{44}, \tag{4.11}$$

$$\Pi_T = \frac{1}{2}(\Pi_{\mu\mu} - \Pi_L), \tag{4.12}$$

where $K_\mu = (k_4, \vec{k}) = (-\omega_n, \vec{k})$ and the propagator is given in the HTL approximation by,

$$D_{\mu\nu}(K) = \frac{(P_T)_{\mu\nu}}{K^2 + \Pi_T} + \frac{(P_L)_{\mu\nu}}{K^2 + \Pi_L} + \alpha \frac{K_\mu K_\nu}{K^4}, \tag{4.13}$$

with $(P_T)_{\mu\nu}$ and $(P_L)_{\mu\nu}$ being respectively the transversal and longitudinal projectors.

The $\mu = \nu$ component of the total self-energy (4.10) is given by,

$$\begin{aligned} i\Pi^{(2)}_{\mu\mu\,Total}(k) &= 4g^2N\int\frac{d^dp}{(2\pi)^d}\frac{1}{p^2} \\ &= 4g^2NT\sum_n\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega_n^2+p^2} \\ &= \frac{-2}{2i\pi}\int\frac{d^3p}{(2\pi)^3}\int_{-i\infty+\delta}^{i\infty+\delta}dp_0\left(\frac{4g^2Nn_B(p_0)}{p_0^2+p^2}\right) \\ &= \frac{4g^2N}{2\pi^2}\int_0^\infty dp\left(\frac{p}{e^{\beta p}-1}\right) \\ &= \frac{g^2NT^2}{3}. \end{aligned} \tag{4.14}$$

where we considered that the dimension is $\delta_{\mu\mu} = d = 4$, $n_B(p) = \frac{1}{e^{\beta p}-1}$ and $\beta = \frac{1}{T}$.

The $(\mu,\nu) = (4,4)$ component of the total self-energy (4.10) is given by,

$$\begin{aligned} i\Pi^{(2)}_{44\,Total}(k) &= g^2N\int\frac{d^dp}{(2\pi)^d}\left(\frac{-6}{(p-k)^2}+\frac{4(\vec{p}\cdot\vec{p})}{p^2(p-k)^2}\right) \\ &= g^2N\int\frac{d^dp}{(2\pi)^d}\left(\frac{-6}{p^2}+\frac{4(\vec{p}\cdot\vec{p})}{p^2(p-k)^2}\right). \end{aligned} \tag{4.15}$$

where we used that $\delta_{44} = -1$, $p_4^2 = -\omega^2$ and $\omega^2 = -p^2 + \vec{p}\cdot\vec{p}$. Here we have to take into account that,

$$\begin{aligned} I &= T\sum_n\int\frac{d^3p}{(2\pi)^3}\left(\frac{\vec{p}\cdot\vec{p}}{(\omega_n^2+p^2)((\omega_n-\omega_l)^2+(p-k)^2)}\right) \\ &= \frac{-1}{8\pi^2}\int_0^\infty dp\left[p^2\int_{-1}^{1}dy\left(\frac{y}{y+\frac{i\omega_l}{k}}\frac{\partial n_B(p)}{\partial p}-\frac{1}{p}n_B(p)\right)\right] \\ &= \frac{T^2}{24}\left(3-\frac{i\omega_l}{k}\log\frac{\frac{i\omega_l}{k}+1}{\frac{i\omega_l}{k}-1}\right) \end{aligned} \tag{4.16}$$

Thus, eq.(4.15) becomes,

$$
\begin{aligned}
i\Pi^{(2)}_{44\ Total}(k) &= \frac{-g^2NT^2}{2} + \frac{g^2NT^2}{6}\left(3 - \frac{i\omega_l}{k}\log\frac{\frac{i\omega_l}{k}+1}{\frac{i\omega_l}{k}-1}\right) \\
&= \frac{g^2NT^2}{6}\left(-\frac{i\omega_l}{k}\log\frac{\frac{i\omega_l}{k}+1}{\frac{i\omega_l}{k}-1}\right). \qquad (4.17)
\end{aligned}
$$

So with the eqs. (4.14) and (4.17) we find that the longitudinal and transversal sectors are given in terms of the temperature T by,

$$
\Pi_L = \frac{g^2NT^2}{3}\frac{K^2}{k^2}\left(-\frac{i\omega_l}{2k}\log\frac{\frac{i\omega_l}{k}+1}{\frac{i\omega_l}{k}-1}\right), \qquad (4.18)
$$

$$
\Pi_T = \frac{g^2NT^2}{6}\left[1 - \frac{K^2}{k^2}\left(-\frac{i\omega_l}{2k}\log\frac{\frac{i\omega_l}{k}+1}{\frac{i\omega_l}{k}-1}\right)\right]. \qquad (4.19)
$$

Making the analytic continuation $i\omega \to \omega + i\delta$, with the Minkowski four-momentum, $K_\mu = (\omega, \vec{k})$, we find the propagator,

$$
D_{\mu\nu}(K) = -\frac{(P_T)_{\mu\nu}}{K^2 - \Pi_T} - \frac{(P_L)_{\mu\nu}}{K^2 - \Pi_L} - \alpha\frac{K_\mu K_\nu}{K^4}, \qquad (4.20)
$$

with,

$$
\Pi_L = \frac{g^2NT^2}{3}\frac{K^2}{k^2}\left(1 + i\pi\Theta\left(1 - \frac{\omega}{k}\right) - \frac{\omega}{2k}\log\frac{\frac{\omega}{k}+1}{\frac{\omega}{k}-1}\right), \qquad (4.21)
$$

$$
\Pi_T = \frac{g^2NT^2}{6}\left[1 - \frac{K^2}{k^2}\left(1 + i\pi\Theta\left(1 - \frac{\omega}{k}\right) - \frac{\omega}{2k}\log\frac{\frac{\omega}{k}+1}{\frac{\omega}{k}-1}\right)\right], \qquad (4.22)
$$

and,

$$
\begin{aligned}
(P_T)_{\mu\nu} &= \delta^i_\mu\delta^j_\nu\left(\delta_{ij} - \frac{k_ik_j}{\vec{k}^2}\right), \\
(P_L)_{\mu\nu} &= \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - (P_T)_{\mu\nu}. \qquad (4.23)
\end{aligned}
$$

Using the series expansion $\frac{\omega}{2k}\log\frac{\frac{\omega}{k}+1}{\frac{\omega}{k}-1} = 1+\frac{k^2}{3\omega^2}+O(\frac{1}{\omega^3})$ and having that for $(1-\frac{\omega}{k})<0 \rightarrow \Theta(1-\frac{\omega}{k})=0$ we find that, after taking the limit $\frac{\omega}{k}\rightarrow\infty$,

$$\begin{aligned}\Pi_L(\omega,k=0) &= \Pi_T(\omega,k=0) = \frac{g^2NT^2}{9} = \frac{\omega_D^2}{3} \equiv \omega_{pl}^2,\\ D_{L,T}(\omega,k=0) &= \frac{1}{\omega^2-\omega_{pl}^2},\end{aligned} \quad (4.24)$$

with $\omega_D^2 = \frac{g^2NT^2}{3}$ being the Debye screening mass, ω_{pl} is the plasma frequency and $D_{L,T}(K) \equiv (K^2-\Pi_{L,T}(K))^{-1}$ being the form factor of the longitudinal and transversal sectors of the gluon propagator at finite temperature. This means that both the longitudinal and transverse non-static gluon propagators in the hot plasma oscillate with a characteristic $\omega_{pl} = \frac{\omega_D}{\sqrt{3}}$ frequency, which, from now on, we will call simply as the plasma mass '$M \equiv \omega_{pl}$'.

With the results of eq.(4.24) we are now able to build an effective action for the description of the Yang-Mills theory at finite temperatures that is able to obtain an identical result for the gluon propagator. Basically, all we need to do is to add a term $\frac{1}{2}M^2A_\mu^aA_\mu^a$ into the action (1.47), in such a way that,

$$S = S_{YM} + S_{FP}^{LC} + \int d^4x\left(\frac{1}{2}M^2A_\mu^{h,a}A_\mu^{h,a}\right), \quad (4.25)$$

From such action, it is easy to see that the gluon propagator will be,

$$\langle A_\mu^a(k)A_\nu^b(-k)\rangle = \left[\frac{1}{k^2+M^2}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\alpha\frac{k_\mu k_\nu}{k^4}\right]\delta^{ab}. \quad (4.26)$$

4.4 The effective thermal massive model within the Gribov-Zwanziger framework

4.4.1 Effective formulation 1

As we saw at the end of section 4.3, in order to study the Yang-Mills theory at finite temperatures, one can build an effective action with which we are able to recover the gluon propagator (4.20). To do so, we just need to add a term of the kind $\frac{1}{2}M^2A^a_\mu A^a_\mu$ into the action of the theory. Then, for the RGZ case, our effective action at finite temperatures becomes,

$$S = S_{RGZ} + S_M, \tag{4.27}$$

where,

$$S_M = \int d^4x \left(\frac{1}{2} M^2 A^a_\mu A^a_\mu \right). \tag{4.28}$$

is the effective term that adds the thermal mass contribution into the theory and $M^2 = \frac{g^2NT^2}{9}$ is the plasma mass.

So the quadratic part over the fields of eq.(4.27) will be,

$$\begin{aligned} S^{quadr} &= \int d^4x \left\{ \frac{1}{2} A^a_\mu \left[k^2\delta_{\mu\nu} - \left(1-\frac{1}{\alpha}\right) k_\mu k_\nu + M^2\delta_{\mu\nu} + \right.\right. \\ &+ \left.\left. \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\left(m^2 + \frac{2Ng_0^2\gamma^4}{k^2+\mu^2}\right) \right] \delta^{ab} A^b_\nu \right\}, \end{aligned} \tag{4.29}$$

from which we obtain the that the gluon propagator is given by,

$$
\begin{aligned}
\langle A^a_\mu(k)A^b_\nu(-k)\rangle &= \Bigg[\frac{k^2+\mu^2}{(k^2+m^2+M^2)(k^2+\mu^2)+2Ng_0^2\gamma^4}\times \\
&\times \left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{\alpha M^2+k^2}\right)+\frac{\alpha k_\mu k_\nu}{k^2\left(\alpha M^2+k^2\right)}+ \\
&- \frac{\alpha k_\mu k_\nu M^2\left(k^2+\mu^2\right)}{k^2\left(\alpha M^2+k^2\right)\left[\left(k^2+M^2+m^2\right)\left(k^2+\mu^2\right)+2Ng_0^2\gamma^4\right]}\Bigg]\delta^{ab},
\end{aligned}
\tag{4.30}
$$

where we can easily see that taking $\alpha \to 0$ we recover the Landau result. In the same way, if we take $M \to 0$ we recover the 0-temperature result.

As it can be observed, the thermal mass M^2 modifies not only the transverse sector of the propagator but also the longitudinal one, since the effective term S_M of the action is given in terms of the fields A^a_μ and not of the non-local transverse fields $A^{h,a}_\mu$. Therefore, by taking the limit $\gamma \to 0$ on the propagator (4.30) we can easily see that it does not reobtain the same propagator of eq. (4.20). This means that the present approach is not the most suitable in order to describe the gluon infrared behaviour at finite temperatures. Then, because of this, we will try in the next section a slightly different approach.

4.4.2 Effective formulation 2

Differently from the previous section, in order to study the YM theory at finite temperatures, we will add to the RGZ action an effective term of the kind $\frac{1}{2}M^2A^{h,a}_\mu A^{h,a}_\mu$. Here the $A^{h,a}_\mu$ field is the non-local, transverse, and gauge invariant field given as an infinite non-local series presented on eq. (2.25). This approach has the advantage of maintaining the gauge invariance of the theory.

As well in the previous case, we have that the total action of this effective model will be,

$$S = S_{RGZ} + S_M, \tag{4.31}$$

but with,

$$S_M = \int d^4x \left(\frac{1}{2} M^2 A^{h,a}_\mu A^{h,a}_\mu \right). \tag{4.32}$$

being the effective term that adds the thermal mass contribution into the theory. Just like before, $M^2 = \frac{g^2 N T^2}{9}$ is the plasma mass.

So the quadratic part of eq.(4.31) will be,

$$\begin{aligned} S^{quadr} &= \int d^4x \Bigg\{ \frac{1}{2} A^a_\mu \Bigg[k^2 \delta_{\mu\nu} - \left(1 - \frac{1}{\alpha}\right) k_\mu k_\nu + \\ &+ \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \left(M^2 + m^2 + \frac{2N g_0^2 \gamma^4}{k^2 + \mu^2}\right) \Bigg] \delta^{ab} A^b_\nu \Bigg\}, \end{aligned} \tag{4.33}$$

from which we obtain the that the gluon propagator is given by,

$$\begin{aligned} \langle A^a_\mu(k) A^b_\nu(-k) \rangle &= \Bigg[\frac{k^2 + \mu^2}{(k^2 + m^2 + M^2)(k^2 + \mu^2) + 2N g_0^2 \gamma^4} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \\ &+ \alpha \frac{k_\mu k_\nu}{k^4} \Bigg] \delta^{ab}. \end{aligned} \tag{4.34}$$

By taking $\alpha \to 0$ we can see that the propagator (4.34) recovers the Landau result. In the same way, if we take $M \to 0$ we recover the 0-temperature result.

Differently from eq. (4.30), the propagator (4.34) does not have its longitudinal sector affected by the thermal mass M, but only the transverse one. This is due to the transversality property of the field $A^{h,a}_\mu$. Then, taking the limit $\gamma \to 0$ we see that the propagator (4.30) reobtains the propagator of eq. (4.20). This means that this second effective formulation is a good candidate for the description of the infrared behaviour of gluons at finite temperatures and, because of this, it will be the model that we will consider in our analysis along the next sections.

4.5 Analysis of the poles of the propagator

Having obtained the gluon propagator (4.34) we need now to analyse its physical spectrum in order to identify the confining and deconfining regimes of the theory. To do so, we will analyze its poles, which will give us the masses of the gluonic degrees of freedom. Then, depending on the value of such masses, we will know if they are associated to physical or unphysical degrees of freedom. Basically, if a pole is real and positive, it will be related to a physical degree of freedom. Otherwise, if it is negative or complex, it will be associated to an unphysical degree of freedom. Ultimately we will interpret the physical degrees of freedom as belonging to the deconfined regime, while the unphysical ones will belong to the confined regime.

This is so due to the fact that immaginary poles lead to the positivity violation of the spectral density function of the Källén–Lehmann representation, which is interpreted as an evidence that their associated degrees of freedom are in a confined regime, since in such regime we do not expect to observe asymptotically free states, as it can be seen on [53; 100].

With this in mind, lets rewrite the gluon propagator (4.34) in the following way,

$$\begin{aligned}\langle A^a_\mu(k)A^b_\nu(-k)\rangle &= \left[-\frac{\frac{\mu^2+u_1}{k^2-u_1}-\frac{\mu^2+u_2}{k^2-u_2}}{\sqrt{\left(-\mu^2+m^2+M^2\right)^2-8Ng_0^2\gamma^4}}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\right.\\ &+ \left.\alpha\frac{k_\mu k_\nu}{k^4}\right]\delta^{ab},\end{aligned} \tag{4.35}$$

where,

$$\begin{aligned} u_1 &= -\frac{1}{2}\left(\sqrt{\left(-\mu^2+m^2+M^2\right)^2-8Ng_0^2\gamma^4}+\mu^2+m^2+M^2\right),\\ u_2 &= \frac{1}{2}\left(\sqrt{\left(-\mu^2+m^2+M^2\right)^2-8Ng_0^2\gamma^4}-\mu^2-m^2-M^2\right). \end{aligned} \tag{4.36}$$

We can see that,

- For $\left(-\mu^2+m^2+M^2\right)^2 < 8Ng_0^2\gamma^4$, or $-2\sqrt{2N}g_0\gamma^2+\mu^2-m^2 < M^2 < 2\sqrt{2N}g_0\gamma^2+\mu^2-m^2$ both poles will be complex, which means that we have a regime of confinement for all the degrees of freedom.

- For $\left(-\mu^2+m^2+M^2\right)^2 \geq 8Ng_0^2\gamma^4$, or $2\sqrt{2N}g_0\gamma^2+\mu^2-m^2 < M^2 < -2\sqrt{2N}g_0\gamma^2+\mu^2-m^2$ we will have two real poles, but only one of them will be positive, which means that we will have only one deconfined degree of freedom. Then we can say that we have a regime of partial confinement-deconfinement.

- If there is no solution for the gap equation, the only consistent choice for the Gribov mass parameter is $\gamma = 0$ leading to a free gluon propagator (deconfined phase).

As $M^2 = \frac{g^2NT^2}{9}$ the relations above give us the critical temperatures in which each one of the three regimes occur.

Comparing to the paper of Canfora [81] we find out that our results reobtain his ones in the limit of $m, \mu \to 0$.

It also is important to state that if we make the calculation of the longitudinal and transverse thermal masses taking into account the Gribov parameter and the condensate masses, we will find a more complex pole structure and probably more confining and deconfining regimes. However, we will leave this discussion for later.

4.6 The RGZ Gap equation in Linear covariant gauges

Another way to analise the phase transitions of the theory is by studying the RGZ gap-equation at finite temperatures. This is so due to the fact that from it we can calculate the relationship between the Gribov parameter γ^4 and the temperature T. Then, if we make use of a consistent procedure, its solutions should be, in the limit $T \to 0$, the same of the zero-temperature solution. In other words, we should obtain solutions that describe confined gluons in the limit of $T \to 0$ and for sufficiently high temperatures we should obtain no solution at all for the gap-equation, meaning that we would be at a regime of freely propagating gluons.

In order to obtain the gap equation, we will follow again the steps of [71], that we used to derive the previous gap equation (2.13), where we obtain that,

$$1 = \frac{3}{4V} N g_0^2 \sum_q \frac{1}{(q^2+\mu^2)\left(q^2+M^2+m^2+\frac{\gamma^4}{(q^2+\mu^2)}\right)} + \frac{1}{4V(N^2-1)\gamma^4}, \tag{4.37}$$

which can be rewritten as,

$$
\begin{aligned}
1 &= \frac{3Ng^2\lambda}{16\pi^3}\sum_n \int_0^1 dR \Bigg\{ R^2 \Bigg[(R^2+\theta_n^2)^2 + \left(m^2+\mu^2+\frac{Ng^2\lambda^2}{36\pi^2}\right)(R^2+\theta_n^2) + \\
&+ \mu^2\left(m^2+\frac{Ng^2\lambda^2}{36\pi^2}\right) + \Gamma^4 \Bigg]^{-1} \Bigg\}, \qquad (4.38)
\end{aligned}
$$

where we used that the thermal mass is given by

$$
M^2 = \frac{Ng^2T^2}{9} = \frac{Ng^2\lambda^2}{36\pi^2}, \qquad (4.39)
$$

and we used the following parametrizations,

$$
\begin{aligned}
R &= \frac{r}{\Lambda}, \\
\lambda &= \frac{2\pi T}{\Lambda}, \\
\theta_n &= \frac{\omega_n}{\Lambda} = n\lambda, \\
\Gamma &= \frac{(2Ng_0^2)^{1/4}\gamma}{\Lambda}. \qquad (4.40)
\end{aligned}
$$

We can see that the gap equation is still the same as the Landau one. This means that the gap equation do not depend on the gauge parameter α. Moreover, after solving the summation,

$$
\mathcal{S} = \sum_n \frac{1}{\left((R^2+\theta_n^2)^2 + (m^2+\mu^2+\frac{Ng^2\lambda^2}{36\pi^2})(R^2+\theta_n^2) + \mu^2(m^2+\frac{Ng^2\lambda^2}{36\pi^2}) + \Gamma^4\right)}, \qquad (4.41)
$$

of eq. (4.38), we obtain that,

$$
\begin{aligned}
\mathcal{S} &= \frac{\pi}{2\lambda\sqrt{\frac{1}{4}\left(\mu^4-4\Gamma^4\right)+\frac{g^4\lambda^4N^2}{5184\pi^4}+\frac{1}{72}m^2\left(\frac{g^2\lambda^2N}{\pi^2}-36\mu^2\right)-\frac{g^2\lambda^2\mu^2N}{72\pi^2}+\frac{m^4}{4}}}\times\\
&\times \Bigg[\coth\Bigg[\frac{\pi}{\lambda}\Bigg(\frac{g^2\lambda^2N}{72\pi^2}+\frac{1}{2}\left(\mu^2+m^2+2R^2\right)-\Bigg(\frac{1}{4}\left(\mu^4-4\Gamma^4\right)+\\
&+ \frac{g^4\lambda^4N^2}{5184\pi^4}+\frac{1}{72}m^2\left(\frac{g^2\lambda^2N}{\pi^2}-36\mu^2\right)-\frac{g^2\lambda^2\mu^2N}{72\pi^2}+\frac{m^4}{4}\Bigg)^{\frac{1}{2}}\Bigg)^{\frac{1}{2}}\Bigg]\times\\
&\times \Bigg[\frac{g^2\lambda^2N}{72\pi^2}+\frac{1}{2}\left(\mu^2+m^2+2R^2\right)-\Bigg(\frac{1}{4}\left(\mu^4-4\Gamma^4\right)+\frac{g^4\lambda^4N^2}{5184\pi^4}+\\
&+ \frac{1}{72}m^2\left(\frac{g^2\lambda^2N}{\pi^2}-36\mu^2\right)-\frac{g^2\lambda^2\mu^2N}{72\pi^2}+\frac{m^4}{4}\Bigg)^{\frac{1}{2}}\Bigg]^{-\frac{1}{2}}+\\
&- \coth\Bigg[\frac{\pi}{\lambda}\Bigg(\frac{g^2\lambda^2N}{72\pi^2}+\frac{1}{2}\left(\mu^2+m^2+2R^2\right)+\Bigg(\frac{1}{4}\left(\mu^4-4\Gamma^4\right)+\frac{g^4\lambda^4N^2}{5184\pi^4}+\\
&+ \frac{1}{72}m^2\left(\frac{g^2\lambda^2N}{\pi^2}-36\mu^2\right)-\frac{g^2\lambda^2\mu^2N}{72\pi^2}+\frac{m^4}{4}\Bigg)^{\frac{1}{2}}\Bigg)^{\frac{1}{2}}\Bigg]\times\\
&\times \Bigg[\frac{g^2\lambda^2N}{72\pi^2}+\frac{1}{2}\left(\mu^2+m^2+2R^2\right)+\Bigg(\frac{1}{4}\left(\mu^4-4\Gamma^4\right)+\frac{g^4\lambda^4N^2}{5184\pi^4}+\\
&+ \frac{1}{72}m^2\left(\frac{g^2\lambda^2N}{\pi^2}-36\mu^2\right)-\frac{g^2\lambda^2\mu^2N}{72\pi^2}+\frac{m^4}{4}\Bigg)^{\frac{1}{2}}\Bigg]^{-\frac{1}{2}}\Bigg],
\end{aligned}
\tag{4.42}
$$

which means that $\mathcal{S}$ is a function of R, λ, Γ, m and μ, as follows,

$$
\mathcal{S} \equiv S(R,\lambda,\Gamma,m,\mu) \tag{4.43}
$$

Then, from equations (4.38) and (4.43) we obtain that the gap equation can be written in a simpler form as,

$$F(\lambda, \Gamma, m, \mu) = 1, \tag{4.44}$$

where,

$$F(\lambda, \Gamma, m, \mu) = \frac{3Ng^2\lambda}{16\pi^3} \int_0^1 dR \Bigg(R^2 S(R, \lambda, \Gamma, m, \mu) \Bigg) = 1, \tag{4.45}$$

from which we obtain the Gribov parameter as a function of λ, m, μ and g.

4.7 Analysis of the regimes of the theory

In this section we will make a more detailed numerical analysis of the regimes of the theory and we will obtain the critical temperatures related to such phase transitions. In order to reach our goal, we shall analyse the gap equation (4.45) numerically. Basically we will plot the dependence of the left side of eq. (4.45) in terms of the factors λ, which encode the temperature dependence, and Γ, which encode the Gribov parameter dependence, as we can see on Figure [4.1], where,

$$F(\lambda, \Gamma, m, \mu) = \frac{3Ng^2\lambda}{16\pi^3} \int_0^1 dR \Bigg(R^2 S(R, \lambda, \Gamma, m, \mu) \Bigg), \tag{4.46}$$

with the coupling constant g being given in the hard thermal regime ($T >> 1$) by,

$$g^2(\lambda) = \frac{8\pi^2}{11 \ln\left(\frac{2\pi T}{\Lambda_{QCD}}\right)} = \frac{8\pi^2}{11 \ln(\alpha\lambda)}, \tag{4.47}$$

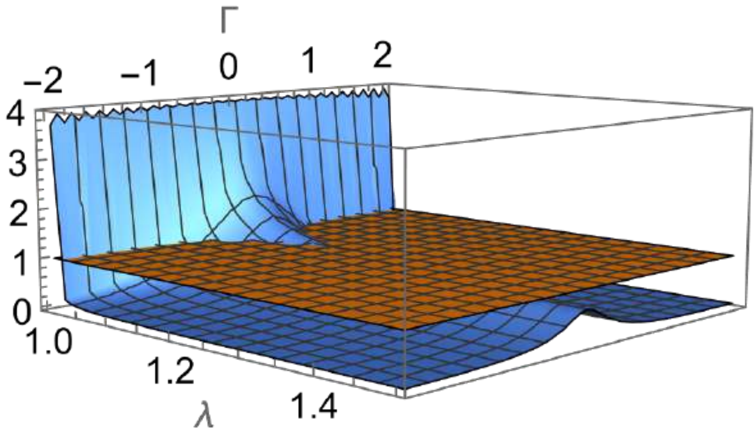


Figure 4.1: Plot of the surface F for different values of λ and Γ. Here we considered $m = 0.5$ and $\mu = 0.1$. The intersection with the plane $F = 1$ occurs for λ below the critical value $\lambda_c^{(1)} = 1.048$. We might emphasize here that we took $\alpha = 1$, but the qualitative behaviour of the gluon propagator does not depend on α.

and,

$$\alpha \equiv \frac{\Lambda}{\Lambda_{QCD}}. \tag{4.48}$$

In this image we can see that the existence of the solution of the gap equation, which is represented by the plane given by $F(\lambda, \Gamma, m, \mu) = 1$, depends on the temperature T.

Thus, we can see from figure [4.2] that the critical temperature occur when $\lambda_c^{(1)} = 1.048$, which means at,

$$\frac{T_c^{(1)}}{\Lambda_{QCD}} = 0.1668, \tag{4.49}$$

when we consider the masses of the condensates fixed with values $m = 0.5$ and $\mu = 0.1$. However when we turn off such massive terms we can see from figure

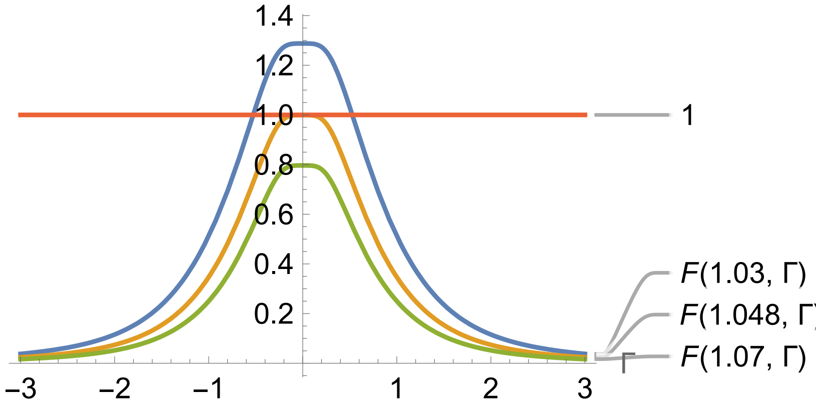


Figure 4.2: Plot of the surface $F(\lambda, \Gamma)$ when $\lambda = 1.03$, $\lambda = 1.048$, $\lambda = 1.07$. Here we considered $m = 0.5$ and $\mu = 0.1$. The red line is where plane $F = 1$ intercept the blue surface of figure [4.1].

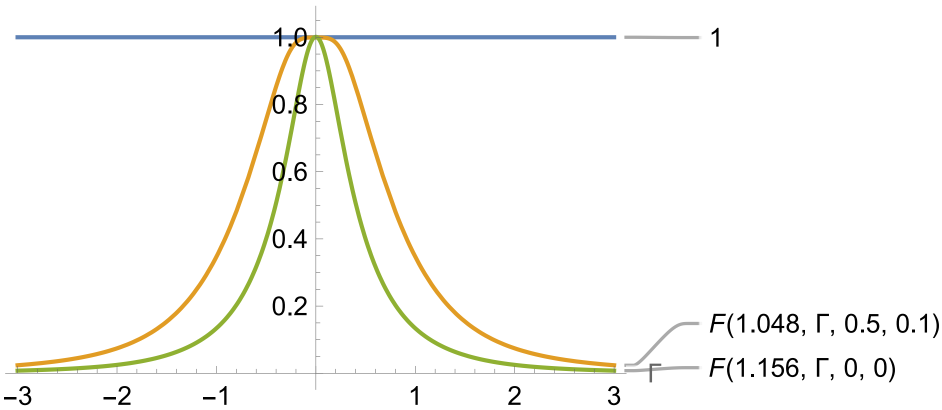


Figure 4.3: Comparison of the plot of the surface $F(\lambda, \Gamma, m, \mu)$ for the case when the masses of the condensates are $m = 0.5$ and $\mu = 0.1$ with the case in which they are null.

[4.3] that the critical temperature becomes,

$$\frac{T_c^{(1)}}{\Lambda_{QCD}} = 0.1840, \tag{4.50}$$

meaning that the masses of the condensates have the the effect of lowering the temperature of phase transition.

Now, the three regimes can be read as follow:

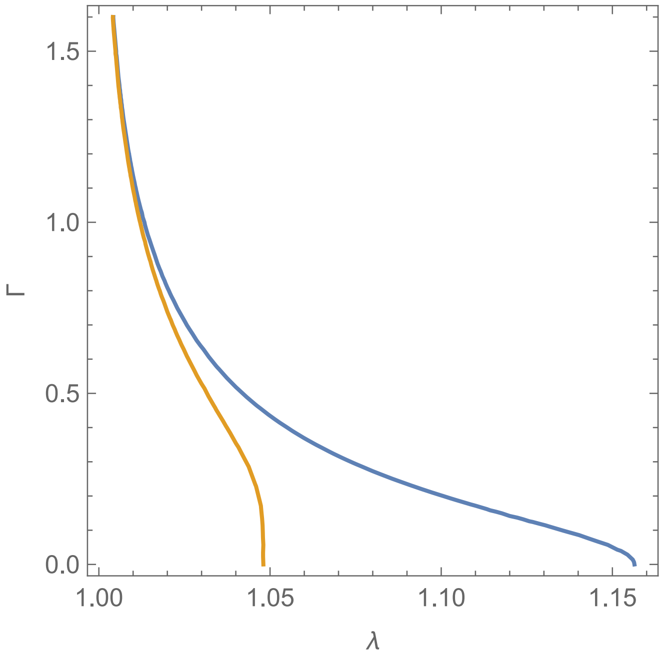


Figure 4.4: Plot of $\Gamma(\lambda)$. Here we considered $m = 0.5$ and $\mu = 0.1$ for the orange curve. It can be seen that Γ reaches zero when λ reaches the critical value $\lambda_c^{(1)} = 1.048$. Yet the blue curve represents the case in which the masses of the condensates are null. For this case Γ reaches zero when λ reaches the critical value $\lambda_c^{(1)} = 1.156$.

- When $T > T_c^{(1)}$ (or $\lambda > \lambda_c^{(1)}$) there is no solution for the gap equation, meaning that in this regime the massive Gribov parameter is null and all the gluonic degrees of freedom are asymptotically free.

- When $T < T_c^{(1)}$ the solutions of the gap equation are generated, in such a way that the Gribov parameter γ is defined. In this regime, as we can see from figure [4.4] the Γ parameter decreases to zero while the temperature increases. Actually this happens when $\lambda_c^{(1)} = 1.048$ in the RGZ case and when $\lambda_c^{(1)} = 1.1840$ GZ one. Moreover, even existing a solution for the gap equation, the total confinement of the propagator will only happen when the discriminant $4\Gamma^4 - \left(-\mu^2 + m^2 + M^2\right)^2$ of eq. (4.36) changes its signal. It is so because, as we can see the from figure [4.5] and eq. (4.36), for

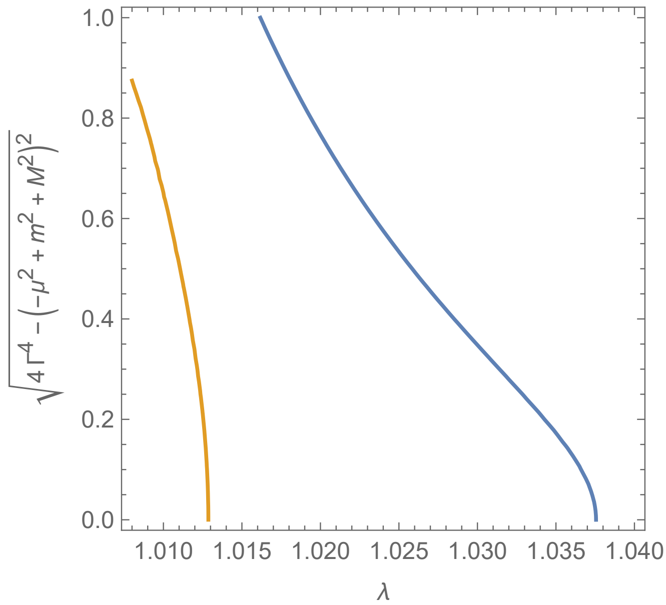


Figure 4.5: Plot of $\sqrt{4\Gamma^4 - (-\mu^2 + m^2 + M^2)^2}$ in terms of λ. We can see here that when $m = 0.5$ and $\mu = 0.1$ there is a phase transition at $\lambda_c^{(2)} = 1.0128$ from a semi-confined to a confined phase. In other words, we can say that the critical temperature of such a phase transition is $\frac{T_c^{(2)}}{\Lambda} = 0.1612$. However, when the masses of the condensates are turned off, the phase transition occurs at $\lambda_c^{(2)} = 1.0375$ and the critical temperature becomes $\frac{T_c^{(2)}}{\Lambda} = 0.1651$

$\lambda_c^1 > \lambda > \lambda_c^2$, with $\lambda_c^2 = 1.0128$ for the RGZ case and $\lambda_c^{(2)} = 1.0375$ for the GZ one, we have a partially confined phase, with one confined and one deconfined (physical) degrees of freedom.

- When $T < T_c^2$, with $\frac{T_c^{(2)}}{\Lambda_{QCD}} = 0.1612$, we have a completely confined phase in the RGZ case, while the same happens at $\frac{T_c^{(2)}}{\Lambda_{QCD}} = 0.1651$ for the GZ one.

It is important to state that from a qualitative point of view, our results are in complete agreement with the ones obtained in [81] by considering only the Gribov copies in the Landau gauge. The only difference is that in our results the critical temperatures are reduced do to the fact that we are also considering the existence of the masses m and μ.

4.8 The infrared regime

After having analysed the hard thermal regime, we need now to make the same work for the infrared one, which is defined by $\lambda < 1$. In order to do so, we need to generalyse the coupling constant g to the infrared regime, what is made in a consistent way by,

$$g^2(g_0, \lambda) = \frac{g_0^2}{1 + \frac{11}{16\pi^2} g_0^2 \ln\left(1 + \alpha^2\lambda^2\right)}, \tag{4.51}$$

since the mass M^2 goes to zero and the coupling constant g goes to g_0 when the temperature T is reduced to zero too, allowing us to reobtain the zero-temperature results.

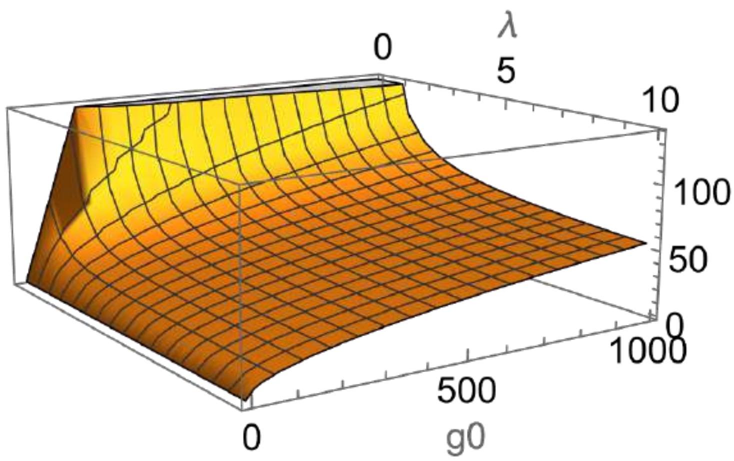


Figure 4.6: Plot of $g(g_0, \lambda)$.

Let us note that for large g_0 the behavior of $g(g_0, \lambda)$ becomes insensible to small variations of g_0 itself; see figure [4.6]. This is consistent with the fact that in quantum field theory bare quantities are infinite but unobservable and they need to be renormalized.

As well as on section (4.7) we will analyse numerically the gap equation, which will be given in this case by,

$$G(g_0, \lambda, \Gamma) = \frac{3g^2 N\lambda}{16\pi^3} \int_0^1 dR(R^2 S(R, g_0, \lambda, \Gamma)). \tag{4.52}$$

Choosing $g_0 = 1000$ we find the same qualitative behaviour that we found in section 4.7, with the same phase transitions, as we can see from figure [4.7]. However, the critical temperatures this time will be given by,

- For the transition from the deconfined to the semi-confined regime,

$$\lambda_c^{(1)} = 0.628 \rightarrow \frac{T_c^{(1)}}{\Lambda_{QCD}} = 0.0999 \approx 0.1, \tag{4.53}$$

  for $m = 0.5$ and $\mu = 0.1$, while,

$$\lambda_c^{(1)} = 1.17 \rightarrow \frac{T_c^{(1)}}{\Lambda_{QCD}} = 0.1862 \approx 0.1, \tag{4.54}$$

  for $m = \mu = 0$, as we can see from figures [4.8], [4.9] and [4.10].

- For the transition from the semi-confined to the confined regime,

$$\lambda_c^{(2)} = 0.396 \rightarrow \frac{T_c^{(2)}}{\Lambda_{QCD}} = 0.063, \tag{4.55}$$

  when $m = 0.5$ and $\mu = 0.1$, while,

$$\lambda_c^{(2)} = 0.64 \rightarrow \frac{T_c^{(2)}}{\Lambda_{QCD}} = 0.102, \tag{4.56}$$

  for $m = \mu = 0$, as we can see from figure [4.11].

Before concluding, we must emphasize that the qualitative behaviour of our plots do not depend on α. The only thing it would change is that increasing/decreasing the α value, it would increase/decrease the critical tempertures in a completely analogous way to the one showed on [81].

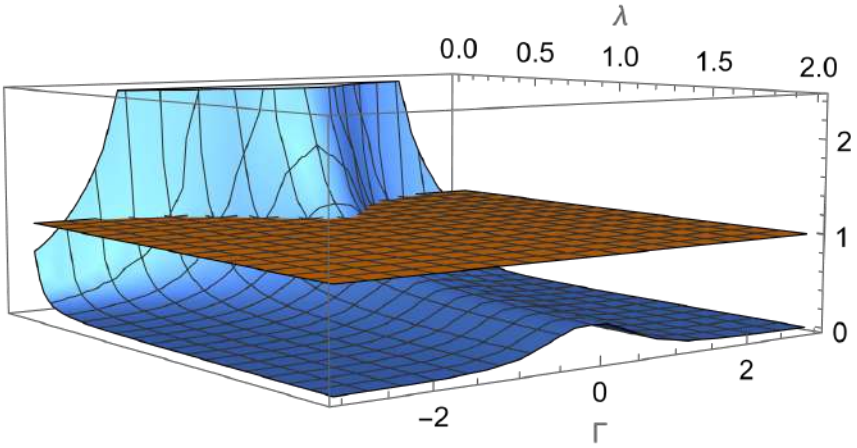


Figure 4.7: Plot of the surface G for different values of λ and Γ. The intersection with the plane $G = 1$ occurs for λ below the critical value $\lambda_c^{(1)} = 0.628$.

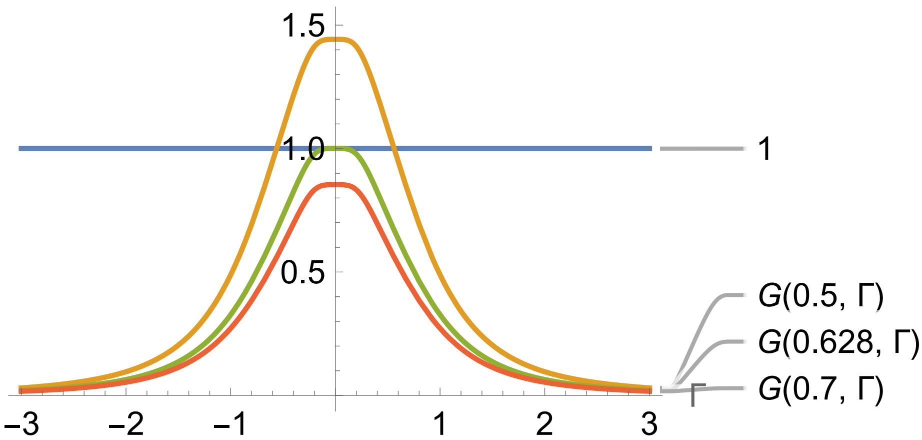


Figure 4.8: Plot of the surface $G(\lambda, \Gamma, m, \mu)$ when $\lambda = 0.5$, $\lambda = 0.628$, $\lambda = 0.7$ (for $m = 0.5$ and $\mu = 0.1$). Here the blue line is where plane $G = 1$ intercept the surface of figure [4.7].

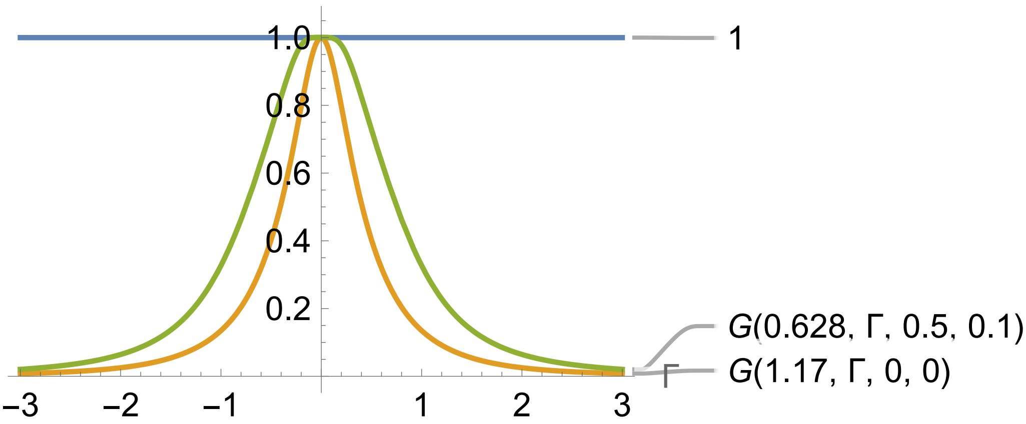


Figure 4.9: Plot of the surface $G(\lambda, \Gamma, m, \mu)$ when $\lambda = 0.628$ (for $m = 0.5$ and $\mu = 0.1$), and $\lambda = 1.17$ (for $m = 0$ and $\mu = 0$).

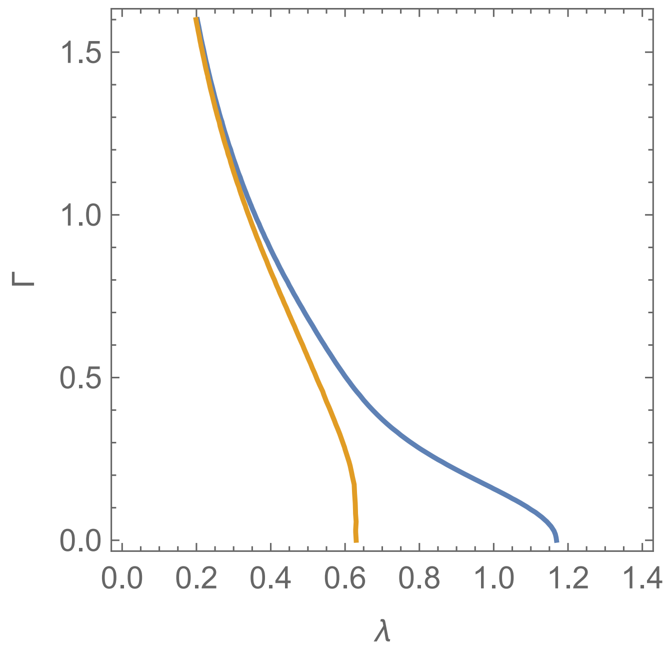


Figure 4.10: Plot of $\Gamma(\lambda)$. Here we can see that Γ reaches zero when λ reaches the critical value $\lambda_c^{(1)} = 0.628$ for $m = 0.5$ and $\mu = 0.1$. Yet for $m = \mu = 0$ it occurs at $\lambda_c^{(1)} = 1.17$

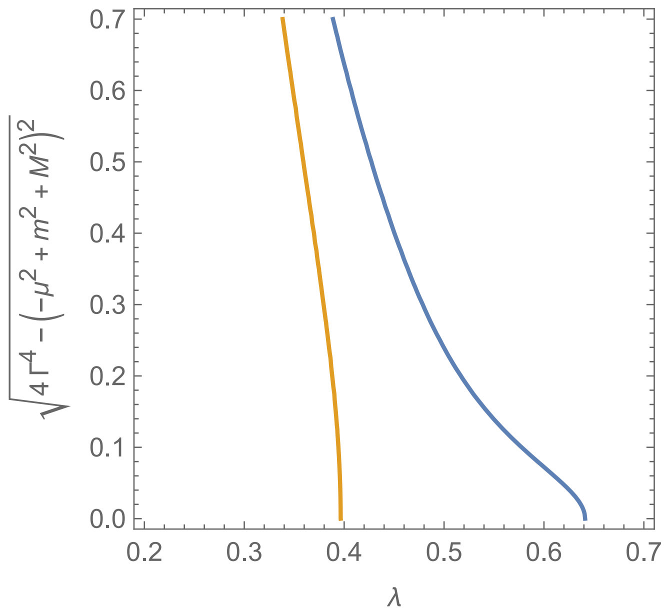


Figure 4.11: Plot of $\sqrt{4\Gamma^4 - (-\mu^2 + m^2 + M^2)^2}$; in terms of λ. We can see here that for $m = 0.5$ and $\mu = 0.1$ there is a phase transition at $\lambda_c^{(2)} = 0.396$ from a semi-confined to a confined phase. In other words, we can say that the critical temperature of such a phase transition is $\frac{T_c^{(2)}}{\Lambda} = 0.063$. However, for $m = \mu = 0$ the phase transition happens at $\lambda_c^{(2)} = 0.64$, which means at the critical temperature $\frac{T_c^{(2)}}{\Lambda} = 0.102$.

4.9 Calculating the thermal mass in terms of the RGZ masses

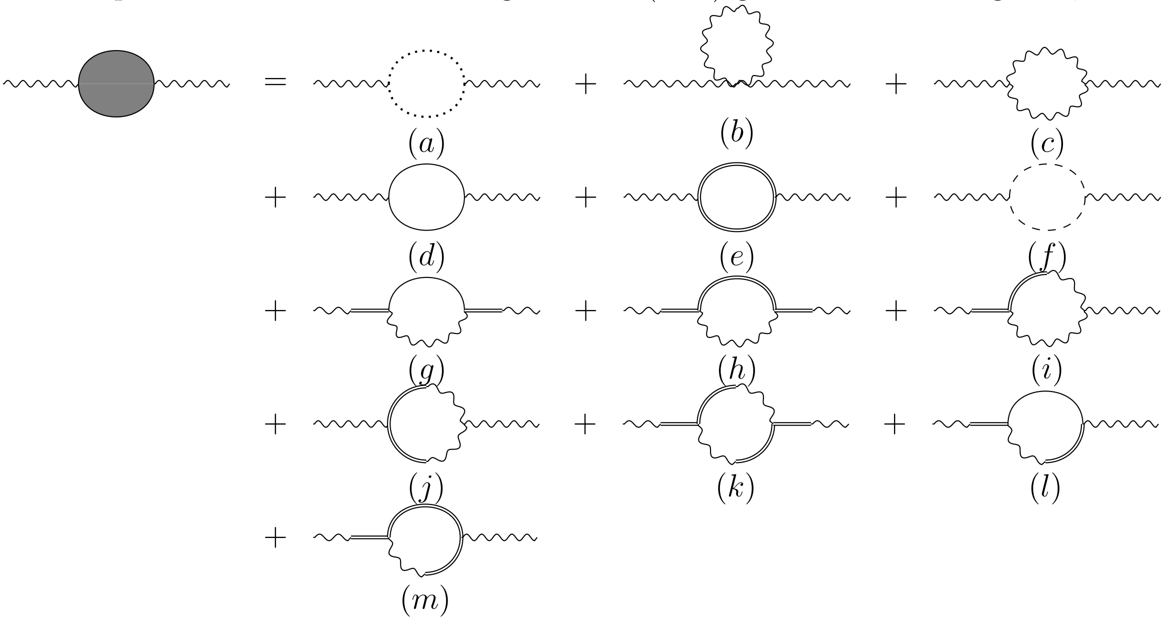
It is important to state that what we did until now was to add a term of the kind $\frac{M^2}{2} A_\mu^a A_\mu^a$ into the Lagrangian of the theory and to calculate how it might modify the physical spectrum of the theory in order to give rise to some phase transition into the theory.

However, as it could be seen, the thermal mass M^2 that we calculated was obtained in therms of the components $\Pi_{44}^{(2)}$ and $\Pi_{\mu\mu}^{(2)}$ of the gluon self energy, which were calculated without considering the existence of the Gribov copies and the masses of the condensates.

Now our goal is to make the same calculation, but considering the diagrams that represent the 2-point functions of the Yang-Mill theory within the Refined Gribov-Zwanziger framework at at 1-loop.

4.9.1 Diagrams

At 1-loop the refined Gribov-Zwanziger action (2.63) gives rise to 13 diagrams,



where,

 (i)

 (ii)

 (iii)

 (iv)

 (v)

 (vi)

are respectively the gluon propagator (i),

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle = \left[\frac{k^2+\mu^2}{((k^2+\mu^2)(k^2+m^2)+2g^2N\gamma^4)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\alpha\frac{k_\mu k_\nu}{k^4}\right]\delta^{ab}, \tag{4.57}$$

the ghost propagator (ii),

$$\langle \bar{c}^a(k)c^b(-k)\rangle = -\frac{\delta^{ab}}{k^2}. \tag{4.58}$$

the bosonic auxiliary field propagator (iii),

$$\begin{aligned}\langle \bar{\phi}^{ab}_\mu(k)\phi^{cd}_\nu(-k)\rangle &= -\frac{g^2\gamma^4 f^{abr}f^{cdr}}{(k^2+\mu^2)((k^2+\mu^2)(k^2+m^2)+2g^2N\gamma^4)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\\ &+ \frac{\delta^{ac}\delta^{bd}\delta_{\mu\nu}}{k^2+\mu^2},\end{aligned} \tag{4.59}$$

the bosonic auxiliary field propagator (iv),

$$\begin{aligned}\langle \phi^{ab}_\mu(k)\phi^{cd}_\nu(-k)\rangle &= -\frac{g^2\gamma^4 f^{abr}f^{cdr}}{(k^2+\mu^2)((k^2+\mu^2)(k^2+m^2)+2g^2N\gamma^4)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right),\\ \langle \phi^{ab}_\mu(k)\phi^{cd}_\nu(-k)\rangle &= \langle \bar{\phi}^{ab}_\mu(k)\bar{\phi}^{cd}_\nu(-k)\rangle,\end{aligned} \tag{4.60}$$

the fermionic auxiliary field propagator (v),

$$\langle \bar{\omega}^{ab}_\mu(k)\omega^{cd}_\nu(-k)\rangle = -\frac{\delta^{ac}\delta^{bd}\delta_{\mu\nu}}{k^2+\mu^2} \tag{4.61}$$

and the mixed propagator (vi),

$$\begin{aligned}\langle \phi^{ab}_\mu(k)A^c_\nu(-k)\rangle &= -\frac{ig\gamma^2 f^{abc}}{((k^2+\mu^2)(k^2+m^2)+2g^2N\gamma^4)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right),\\ \langle \phi^{ab}_\mu(k)A^c_\nu(-k)\rangle &= \langle \bar{\phi}^{ab}_\mu(k)A^c_\nu(-k)\rangle.\end{aligned} \tag{4.62}$$

However, in the high Thermal Limit (HTL) only 7 of them will contribute to the gluon self-energy, while the other 6 will vanish when we take the internal momentum $k \to 0$.

This 7 diagrams in the HTL are:

- The diagram (a) is,

$$i\Pi^{(2)ab}_{\mu\nu}(k) = \int \frac{d^d p}{(2\pi)^d}\left(\frac{g^2 N p^\mu p^\nu \delta^{ab}}{p^2\left(k-p\right)^2}\right). \tag{4.63}$$

- The diagram (b) is,

$$\begin{aligned} i\Pi^{(2)ab}_{\mu\nu}(k) &= \int \frac{d^d p}{(2\pi)^d}\Bigg(-\frac{2g^2 N\left(m^2-p^2\right)\delta^{ab}}{p^2}\times \\ &\times \frac{(p^2(\alpha+d-2)g^{\mu\nu}-(\alpha-1)p^\mu p^\nu)}{\left(2\gamma^4 g^2 N+\mu^2 m^2-p^2\left(\mu^2+m^2\right)+\left(p^2\right)^2\right)}\Bigg). \end{aligned} \tag{4.64}$$

- The diagram (c) is,

$$\begin{aligned} i\Pi^{(2)ab}_{\mu\nu}(k) &= \int \frac{d^d p}{(2\pi)^d}\Bigg(-\frac{2g^2 N\left(m^2-p^2\right)^2\delta^{ab}}{\left(p^2-x_1^2\right)\left(p^2-x_2^2\right)}\times \\ &\times \frac{((-\alpha+2d-2)p^\mu p^\nu+\alpha p^2 g^{\mu\nu})}{((k-p)^2-x_1^2)\left((k-p)^2-x_2^2\right)}\Bigg). \end{aligned} \tag{4.65}$$

- The diagram (d) is,

$$\begin{aligned} i\Pi^{(2)ab}_{\mu\nu}(k) &= \int \frac{d^d p}{(2\pi)^d}\Bigg(-\frac{\gamma^8(d-1)g^6 N^3 p^\mu p^\nu \delta^{ab}}{4\left(p^2-m^2\right)\left(x_1^2-p^2\right)\left(p^2-x_2^2\right)\left((k-p)^2-x_2^2\right)} \\ &\times \frac{1}{((k-p)^2-m^2)\left(-(k-p)^2+x_1^2\right)}\Bigg). \end{aligned} \tag{4.66}$$

- The diagram (e) is,

$$
\begin{aligned}
i\Pi^{(2)ab}_{\mu\nu}(k) &= \int \frac{d^dp}{(2\pi)^d}\Bigg\{g^2N^2p^\mu p^\nu\delta^{ab}\Bigg[8p^2\left(-p^2+x_1^2+x_2^2\right)\times \\
&\times \Bigg(-\left(\gamma^4(d-1)g^2\right)+dp^2\frac{(N^2-1)}{2N}\left(p^2-x_1^2-x_2^2\right)+ \\
&+ 2dx_1^2x_2^2\frac{(N^2-1)}{2N}\Bigg)-8dx_1^4x_2^4\frac{(N^2-1)}{2N}-\left(\gamma^8(d-1)g^4N\right)+ \\
&+ 8\gamma^4(d-1)g^2x_1^2x_2^2\Bigg]\frac{1}{\left(-(k-p)^2+m^2\right)\left((k-p)^2-x_2^2\right)}\times \\
&\times \frac{1}{4\left(m^2-p^2\right)\left(p^2-x_1^2\right)\left(p^2-x_2^2\right)\left((k-p)^2-x_1^2\right)}\Bigg\}, \qquad (4.67)
\end{aligned}
$$

- The diagram (f) is,

$$
i\Pi^{(2)ab}_{\mu\nu}(k) = \int\frac{d^dp}{(2\pi)^d}\left(\frac{dg^2N\left(N^2-1\right)p^\mu p^\nu\delta^{ab}}{\left(m^2-p^2\right)\left(-(k-p)^2+m^2\right)}\right). \qquad (4.68)
$$

- The diagram (j) is,

$$
\begin{aligned}
i\Pi^{(2)ab}_{\mu\nu}(k) &= \int\frac{d^dp}{(2\pi)^d}\left(\frac{\gamma^4(d-1)g^4N^2p^\mu p^\nu\delta^{ab}}{\left(x_1^2-p^2\right)\left(p^2-x_2^2\right)\left(-(k-p)^2+x_1^2\right)}\times\right. \\
&\times \left.\frac{1}{\left((k-p)^2-x_2^2\right)}\right),
\end{aligned}
\qquad (4.69)
$$

where x_1^2 and x_2^2 are given by,

$$
\begin{aligned}
x_1^2 &= \frac{1}{2}\left(\sqrt{-8\gamma^4g^2N+m^4-2m^2\mu^2+\mu^4}+m^2+\mu^2\right), \\
x_2^2 &= \frac{1}{2}\left(-\sqrt{-8\gamma^4g^2N+m^4-2m^2\mu^2+\mu^4}+m^2+\mu^2\right). \qquad (4.70)
\end{aligned}
$$

Summing the diagrams 4.63, 4.64, 4.65 ,4.66, 4.67, 4.68, 4.69 we obtain the gluon self-energy $i\Pi^{(2)ab}_{\mu\nu}$. Then taking the components $i\Pi^{(2)}_{44}$ and $i\Pi^{(2)}_{\mu\mu}$ we obtain both the longitudinal and the transverse thermal masses, respectively Π_L and Π_T, by means of the eqs. (4.11) and (4.12). Then, considering the propagator at 0-temperature (4.57), we can rewrite it as,

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle = \left[\frac{1}{(k^2+\bar{m}^2)}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{\alpha k_\mu k_\nu}{k^4}\right]\delta^{ab}, \tag{4.71}$$

with,

$$\bar{m}^2 = m^2 + \frac{2Ng_0^2\gamma^4}{k^2+\mu^2}, \tag{4.72}$$

which means that if we make the analytic continuation $i\omega \to \omega + i\delta$, with the Minkowski four-momentum, $K_\mu = (\omega, \vec{k})$, we find that the propagator of the theory at finite temperatures will be,

$$\begin{aligned} D_{\mu\nu}(K) &= -\frac{(P_T)_{\mu\nu}}{K^2-\bar{m}^2-\Pi_T} - \frac{(P_L)_{\mu\nu}}{K^2-\bar{m}^2-\Pi_L} - \alpha\frac{K_\mu K_\nu}{K^4}, \\ \bar{m}^2 &= m^2 - \frac{2Ng_0^2\gamma^4}{K^2-\mu^2}, \end{aligned} \tag{4.73}$$

or, in oder words,

$$\begin{aligned} D_{\mu\nu}(K) &= \frac{(-K^2+\mu^2)\,(P_T)_{\mu\nu}}{(-K^2+m^2+\Pi_T)(-K^2+\mu^2)+2Ng_0^2\gamma^4} + \\ &+ \frac{(-K^2+\mu^2)\,(P_L)_{\mu\nu}}{(-K^2+m^2+\Pi_L)(-K^2+\mu^2)+2Ng_0^2\gamma^4} - \alpha\frac{K_\mu K_\nu}{K^4}, \end{aligned} \tag{4.74}$$

with Π_L and Π_T being really complicate functions of m, μ, and γ even after removing all the divergent terms.

Doing so, one important result that we found this time is that the masses Π_L

and Π_T are not the same in the limit $k \to 0$, since they are given by,

$$
\begin{aligned}
\bar{x}_1^2 &= \frac{1}{2}\Bigg(- \sqrt{-8Ng^2\gamma^4 + \mu^4 - 2\mu^2 m^2 + m^4 - 2\mu^2\Pi_L + 2m^2\Pi_L + \Pi_L^2} + \\
&+ \mu^2 + m^2 + \Pi_L \Bigg), \\
\bar{x}_2^2 &= \frac{1}{2}\Bigg(\sqrt{-8Ng^2\gamma^4 + \mu^4 - 2\mu^2 m^2 + m^4 - 2\mu^2\Pi_L + 2m^2\Pi_L + \Pi_L^2} + \\
&+ \mu^2 + m^2 + \Pi_L \Bigg),
\end{aligned}
\tag{4.75}
$$

for the longitudinal sector, and,

$$
\begin{aligned}
\bar{y}_1^2 &= \frac{1}{2}\Bigg(- \sqrt{-8Ng^2\gamma^4 + \mu^4 - 2\mu^2 m^2 + m^4 - 2\mu^2\Pi_T + 2m^2\Pi_T + \Pi_T^2} + \\
&+ \mu^2 + m^2 + \Pi_T \Bigg), \\
\bar{y}_2^2 &= \frac{1}{2}\Bigg(\sqrt{-8Ng^2\gamma^4 + \mu^4 - 2\mu^2 m^2 + m^4 - 2\mu^2\Pi_T + 2m^2\Pi_T + \Pi_T^2} + \\
&+ \mu^2 + m^2 + \Pi_T \Bigg),
\end{aligned}
\tag{4.76}
$$

for the transverse one. This is relevant since, in principle, it prevents us from adding the classic effective term $\frac{M^2}{2}A_\mu^a A_\mu^a$ into the action in order to describe the theory at finite temperatures. However, it is well known that the condensates of the 2-dimensional operators $A_\mu^a A_\mu^a$ and $\bar{\phi}_\mu^{ab}\phi_\mu^{ab} - \bar{\omega}_\mu^{ab}\omega_\mu^{ab}$ only exist in the infrared regime, which means that they might vanish as long as we get closer to the ultraviolet regime.

This would in principle give us back the opportunity to build an effective field theory from which we could obtain a propagator that would emulate the behaviour of the results of eq. (4.74). However, in order to do so, we would need first to obtain the gap eq. of the masses of the condensates and solve them at finite temperatures in order to be able to obtain expressions of the transverse and longitudinal masses of the gluon propagator purely in terms of the temperature T. Then we would be able to determine an even more accurate behaviour than the one obtained in the previous section. However, as this is not a simple task, we will not let this analysis for a future work, where we will also analyse whether the transition between the confining and the deconfining phases occur in such case.

4.10 Conclusions and future perspectives

In this work we studied the Refined Gribov-Zwanziger theory at finite temperature in order to obtain information on the effects of the Gribov parameter and of the masses of the condensates on the pole structure of propagators of the theory and the behaviour, revealing us the phase transitions of the theory due to the variation of the temperature T.

In order to do such kind of analysis we used a semi-classical approach in which we considered an effective massive Yang-Mills theory in order to add the thermal effects into the poles of the gluon propagator. Basically what we did was to add a term $\frac{1}{2}M^2 A_\mu A_\mu$ into action of the theory. Here the massive term M^2 represents the 1-loop finite-temperature corrections of the theory, meaning that it contains all the thermal information. So without this term we would not be able to see any kind of phase transition.

By means of this method we were able to identify two critical temperatures of phase transitions. One that is related to the transition between a purely confined phase to a partially confined one, with a physical degree of freedom and a non-physical one. The other one is related to a phase transition between the partially confined phase to the completely deconfined one.

At the end, we calculated the thermal mass M^2 in terms of the Gribov parameter and the masses of the condensates. From it we showed that there still exist regimes of temperatures for which there are confining and deconfining phases. However do to the complexity of the results, we were not able to write down an effective action from which we could obtain results that could mimic the ones from eq. (4.74), which is something that we intend to do in a future work. In such a work, would be surely interesting to calculate the masses of the condensates by solving their respectively gap equations at finite temperature, in order to be able to analyse the longitudinal and transverse masses Π_L and Π_T purely in terms of the temperature T, which would facilitate a lot our analysis of the phase transitions of the theory.

Chapter 5

The massive Yang-Mills theory

Summary

As it was seen on chapters 2 and 3, the Gribov-Zwanziger quantization procedure in the Maximal Abelian gauge leads to the arising of non-perturbative massive term (Gribov parameter) into the poles of the propagator (2.71) and (2.70) of the Abelian sector of the Yang-Mills theory. Moreover, due to the non-local structure of the Horizon function, which is introduced in order to eliminate the infinitesimal Gribov copies, we are led to introduce two pairs of bosonic and fermionic fields in order to implement a suitable localization of the complete action. However, their own dynamics renders the formation of condensates, which are due to dynamical infrared instabilities of the model leading to the refined Gribov-Zwanziger action (RGZ). Such condensates give massive contributions into the poles of the propagators of both Abelian and non-Abelian sectors of the theory. Consequently their behaviour becomes qualitatively in good agreement to the one obtained on lattice [4; 101] where it was shown their finiteness in the limit $k \to 0$. More than that it was shown that the mass of the non-Abelian propagator is bigger than the Abelian one, which is a lattice evidence of the

Abelian dominance in the deep infrared regime.

With this in mind, in this chapter we will introduce a different effective method to study the Abelian dominance in the non-perturbative regime of the Yang-Mills theory. Basically, the idea is to define an effective massive Yang-Mills theory by adding three massive terms of the kind $\frac{m^2}{2}A^3_\mu A^3_\mu$, $\frac{M^2}{2}A^a_\mu A^a_\mu$ and $-S^2\bar{c}^a c^a$. Such masses will enclosure all the non-perturbative features of the theory and will, at the end, be determined by fitting the form factors of the Abelian and non-Abelian gluon propagators corrected at 1-loop with the lattice results of Bornyakov's paper [4].

5.1 The massive $SU(2)$ Yang-Mills Theory in the Maximal Abelian gauge

We start writing the massive $SU(2)$ Yang-Mills action in the maximal Abelian gauge in the Euclidean space,

$$S = S_{YM} + S_{Mm} + S^{MAG}_{FP} + S_\alpha, \tag{5.1}$$

where, using the Cartan decomposition (1.49) we have,

$$\begin{aligned}
S_{YM} &= \int d^4x \left(\frac{1}{4}F^A_{\mu\nu}F^A_{\mu\nu}\right) = \int d^4x \left(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{4}F^3_{\mu\nu}F^3_{\mu\nu}\right) \\
&= \int d^4x \Bigg[\frac{1}{2}\Big((\partial_\mu A^a_\nu)(\partial_\mu A^a_\nu) - (\partial_\mu A^a_\nu)(\partial_\nu A^a_\mu)\Big) + \frac{1}{2}\Big((\partial_\mu A^3_\nu)(\partial_\mu A^3_\nu) + \\
&- (\partial_\mu A^3_\nu)(\partial_\nu A^3_\mu)\Big) + g f^{ab3}\Big((\partial_\mu A^a_\nu)A^3_\nu A^b_\mu - (\partial_\mu A^a_\nu)A^3_\mu A^b_\nu\Big) + \\
&+ \frac{1}{2}g f^{ab3}\Big((\partial_\mu A^3_\nu)A^a_\mu A^b_\nu - (\partial_\mu A^3_\nu)A^a_\nu A^b_\mu\Big) + \frac{1}{4}g^2 f^{ab3}f^{cd3}A^a_\mu A^b_\nu A^c_\mu A^d_\nu + \\
&+ \frac{1}{2}g^2 f^{ab3}f^{ac3}\Big(A^3_\mu A^b_\nu A^3_\mu A^c_\nu - A^3_\mu A^b_\nu A^3_\nu A^c_\mu\Big)\Bigg],
\end{aligned} \tag{5.2}$$

which is the Yang-Mills action,

$$\begin{aligned}
S^{MAG}_{FP} &= \int d^4x \Bigg[-\frac{1}{\alpha}g f^{ab3}(\partial_\mu A^a_\mu)A^3_\nu A^b_\nu + \frac{1}{2\alpha}(\partial_\mu A^a_\mu)(\partial_\nu A^a_\nu) + \\
&+ \frac{g^2}{2\alpha}f^{ab3}f^{ac3}A^3_\mu A^b_\mu A^3_\nu A^c_\nu + g^2 f^{ad3}f^{bc3}\bar{c}^a A^c_\mu A^d_\mu c^b + g^2 f^{ac3}f^{cb3}\bar{c}^c A^3_\mu A^3_\mu c^b + \\
&- g f^{ab3}(\partial_\mu \bar{c}^3)A^a_\mu c^b + g f^{ab3}\Big((\partial_\mu \bar{c}^a)A^3_\mu c^b - \bar{c}^a A^3_\mu(\partial_\mu c^b)\Big) + \bar{c}^a(\partial_\mu\partial_\mu c^a) + \\
&- \bar{c}^3(\partial_\mu\partial_\mu c^3)\Bigg],
\end{aligned} \tag{5.3}$$

is the Faddeev-Popov action in the maximal Abelian gauge,

$$S_{Mm} = \int d^4x \left(\frac{1}{2}M^2 A^a_\mu A^a_\mu + \frac{1}{2}m^2 A^3_\mu A^3_\mu\right), \tag{5.4}$$

are the effective massive terms whose masses we will fit with the lattice [4], and,

$$S_\alpha = \int d^4x \left(\frac{1}{4}\alpha g^2 f^{ab3}f^{cd3}\bar{c}^a\bar{c}^b c^c c^d + \frac{\alpha}{2}b^a b^a - S^2\bar{c}^a c^a\right), \tag{5.5}$$

are external terms added in order to garantee the renormalizability of this effective massive Yang-Mills theory, as it was shown in [29; 102].

Moreover, we might also mention that besides the terms $\frac{m^2}{2}A^3_\mu A^3_\mu$ and $\frac{M^2}{2}A^a_\mu A^a_\mu$, which generate masses into the poles of the Abelian and non-Abelian gauge field propagators, we also added a term $-S^2\bar{c}^a c^a$ that will give rise to an effective mass S^2 into the pole of the non-Abelian ghost propagator, which is in agreement with the works [47; 61; 64] in the RGZ framework and with [4] in the lattice.

From eq. (5.1) we can calculate the propagators of the theory. The non-Abelian and Abelian propagators of the gauge fields are, respectively, given by,

$$\begin{aligned} \langle A^a_\mu(k)A^b_\nu(-k)\rangle &= \frac{1}{k^2+M^2}\left(\delta_{\mu\nu}-\frac{(1-\alpha)}{k^2+\alpha M^2}k_\mu k_\nu\right)\delta^{ab}, \\ \langle A^3_\mu(k)A^3_\nu(-k)\rangle &= \frac{1}{k^2+m^2}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\delta^{33}, \end{aligned} \tag{5.6}$$

or, in another way, decomposing its transverse and longitudinal parts,

$$\begin{aligned} \langle A^a_\mu(k)A^b_\nu(-k)\rangle &= \frac{1}{k^2+M^2}\left(\delta^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)\delta^{ab}+\left(\frac{\alpha k^\mu k^\nu}{k^2\left(k^2+\alpha M^2\right)}\right)\delta^{ab}, \\ \langle A^3_\mu(k)A^3_\nu(-k)\rangle &= \frac{1}{k^2+m^2}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\delta^{33}, \end{aligned} \tag{5.7}$$

while the non-Abelian and Abelian propagators of the ghost fields are, respectively, given by,

$$\langle \bar{c}^a(k)c^b(-k)\rangle = \frac{1}{k^2+S^2}\delta^{ab}, \tag{5.8}$$

$$\langle \bar{c}^3(k)c^3(-k)\rangle = \frac{1}{k^2}\delta^{33}, \tag{5.9}$$

We can see from eq. (5.7) that the Abelian gluon propagator is purely transverse while the non-Abelian one has a massive longitudinal part that goes to 0 when $\alpha \to 0$ as well as its mass αM^2. Regarding the ghost propagators, we can see that only the non-Abelian ones will obtain an effective mass, since the Abelian ghost sector decouples from the theory, meaning that there is no reason why to consider a model that contains massive Abelian ghosts propagators.

5.2 Vertices

As our main goal in this chapter is to calculate the Abelian and non-Abelian gluon self-energy, in order to be able to write the propagators of the theory with their masses corrected at 1-loop order, we need, first of all, to obtain the vertices of the theory. Yet, in order to calculate the vertices of the theory we will use the Fourier transformation to the momentum space defined in the following form,

$$\Psi(x) = \int \frac{d^4p}{(2\pi)^4} \left(\tilde{\Psi}(p) e^{-ipx} \right). \tag{5.10}$$

Then from the action of eq. (5.1) we obtain the following vertices,

- *A-A-A*

$$\begin{aligned} V^{fh3}_{\alpha\beta\sigma}(q_1, q_2, q_3) &= -ig(2\pi)^4 \delta^4(q_1 + q_2 + q_3) f^{fh3} \Bigg(\delta^{\alpha\beta}(q_1^\sigma - q_2^\sigma) + \\ &+ \delta^{\sigma\beta}(q_2^\alpha - q_3^\alpha) + \delta^{\sigma\alpha}(q_3^\beta - q_1^\beta) - \frac{1}{\alpha}(q_2^\beta \delta^{\alpha\sigma} - q_1^\alpha \delta^{\delta\sigma}) \Bigg). \end{aligned} \tag{5.11}$$

- *A-A-A-A*

$$V^{fhns}_{\alpha\beta\sigma\lambda}(q_1,q_2,q_3,q_4) = (2\pi)^4 g^2 \delta^4(q_1+q_2+q_3+q_4)\Bigg[f^{sn3}f^{hf3}\Big(\delta^{\beta\lambda}\delta^{\alpha\sigma} + - \delta^{\alpha\sigma}\delta^{\beta\sigma}\Big) + f^{fs3}f^{nh3}\Big(\delta^{\sigma\alpha}\delta^{\beta\lambda} - \delta^{\sigma\lambda}\delta^{\beta\alpha}\Big) + + f^{sh3}f^{fn3}\Big(\delta^{\alpha\lambda}\delta^{\sigma\beta} - \delta^{\sigma\lambda}\delta^{\alpha\beta}\Big)\Bigg], \tag{5.12}$$

and,

$$V^{fh33}_{\alpha\beta\sigma\lambda}(q_1,q_2,q_3,q_4) = (2\pi)^4 g^2 \Bigg[f^{ah3}f^{af3}\left(\delta^{\sigma\lambda}\delta^{\alpha\beta} - \delta^{\alpha\lambda}\delta^{\sigma\beta} + \frac{1}{\alpha}\delta^{\beta\lambda}\delta^{\alpha\sigma}\right) + + f^{af3}f^{ah3}\left(\delta^{\sigma\lambda}\delta^{\beta\alpha} - \delta^{\beta\lambda}\delta^{\sigma\alpha} + \frac{1}{\alpha}\delta^{\alpha\lambda}\delta^{\beta\sigma}\right)\Bigg] \times \times \delta^4(q_1+q_2+q_3+q_4). \tag{5.13}$$

- *A-$\bar{c}$-c*

$$V^{3ab}_{\mu}(q_1,q_2,q_3) = -ig(2\pi)^4 f^{ab3}\delta^{\mu\nu}\delta^4(q_1+q_2+q_3)(q_3^\nu - q_2^\nu), \tag{5.14}$$

and,

$$V^{a3b}_{\mu}(q_1,q_2,q_3) = igf^{ab3}q_2^\mu(2\pi)^4\delta^4(q_1+q_2+q_3). \tag{5.15}$$

- *A-A-$\bar{c}$-c*

$$V^{ab}_{\mu\nu}(q_1,q_2,q_3,q_4) = -2g^2\delta^{ab}\delta^{\mu\nu}(2\pi)^4\delta^4(q_1+q_2+q_3+q_4), \tag{5.16}$$

and,

$$V^{gfnh}_{\mu\nu}(q_1, q_2, q_3, q_4) = g^2\delta^{\mu\nu}(2\pi)^4(f^{hf3}f^{gn3} + f^{hn3}f^{gf3})\delta^4(q_1 + q_2 + q_3 + q_4). \tag{5.17}$$

- $\bar{c}$-$\bar{c}$-c-c

$$V^{gfnh}(q_1, q_2, q_3, q_4) = -\alpha g^2(2\pi)^4(f^{nh3}f^{fg3} - f^{ng3}f^{fh3})\delta^4(q_1 + q_2 + q_3 + q_4). \tag{5.18}$$

5.3 Diagrams

In this section we will finally calculate the Abelian and non-Abelian two-point functions of the gauge fields at 1-loop. In order to do so, we will adopt some conventions, as follows;

- We will represent non-Abelian gauge propagators by straight lines, while waved lines will represent the Abelian ones and the dotted lines will represent the non-Abelian ghost propagators.
- For the construction of the vertices, we will give a positive sign + for all the momentum entering on a vertex and a negative sign − for all the momentum exiting from a vertex.

In order to properly solve the diagrams we made use of the Wolfram Mathematica package FeynCalc [103]. With this package we are able to implement a symbolic semi-automatic evaluation of Feynman diagrams and algebraic expressions in quantum field theory. Basically, this package allows us to automatize the process of tensor reduction and partial fractioning of loop integrals. Both of these functionalities are very usefull for us, specially the partial fractioning one. With it we are able to write the diagrams in terms of its master integrals before evaluating them.

However, as all theses intermediate calculation process in order to solve the 1-loop Feynman diagrams is too extensive, we will leave them out of this thesis and leave the reference of [102] for more details. Moreover, as the results of our diagrams are also too wide, we chose to rewrite them in terms of the following functions,

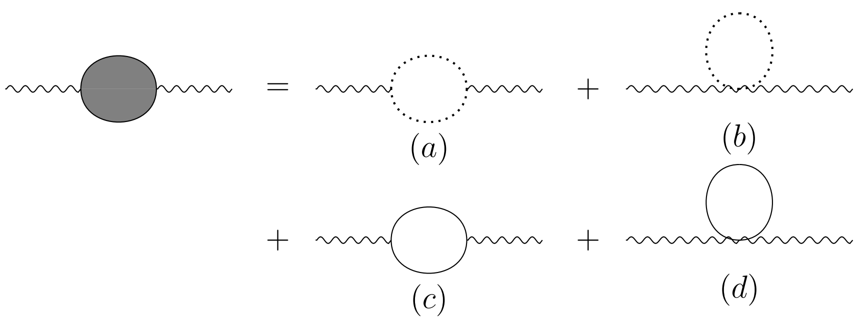
$$\chi(m_1) = \frac{1}{(4\pi)^{\frac{d}{2}}}\Gamma\left(1-\frac{d}{2}\right)(m_1)^{\frac{d}{2}-1}, \tag{5.19}$$

$$\eta(m_1, m_2) = \frac{1}{(4\pi)^{\frac{d}{2}}}\Gamma\left(2-\frac{d}{2}\right)\int_0^1 dx\left(p^2x(1-x)+xm_1+(1-x)m_2\right)^{\frac{d}{2}-2}, \tag{5.20}$$

which will give us a more fancy look to our final solution.

5.3.1 Abelian gluon self-energy

The two-point function of the Abelian sector of the gauge fields is given as the sum of the four diagrams of the figure below.



However, to evaluate these diagrams means in principle to evaluate their self-energy, which is represented by the core of each diagram with its external legs amputated. The total self-energy of the Abelian gluon two-point function will be given by,

$$i\Pi_{33}^{\mu\nu}(k) = i\Pi_{33}^{\mu\nu(a)}(k) + i\Pi_{33}^{\mu\nu(b)}(k) + i\Pi_{33}^{\mu\nu(c)}(k) + i\Pi_{33}^{\mu\nu(d)}(k) \tag{5.21}$$

where,

- $i\Pi_{33}^{\mu\nu(a)}(k)$ is the self-energy of the ghost sunrise diagram (a), being given by,

$$\begin{aligned} i\Pi_{33}^{\mu\nu(a)}(k) &= -g^2 f^{ba3} f^{a'b'3} \int \frac{d^d p}{(2^\pi)^d} \left[\delta^{\mu\alpha} \left(p_\alpha - (-k-p)_\alpha \right) \frac{\delta^{aa'}}{p^2+S^2} \delta^{\beta\nu} \times \right. \\ &\times \left. \left((k+p)_\beta - (-p)_\beta \right) \frac{\delta^{bb'}}{(k+p)^2+S^2} \right] \\ &= -g^2 f^{ba3} f^{ab3} \int \frac{d^d p}{(2^\pi)^d} \left[\frac{(k+2p)^\mu}{p^2+S^2} \frac{(k+2p)^\nu}{(k+p)^2+S^2} \right], \end{aligned} \tag{5.22}$$

  where 1 is the symmetry factor.

- $i\Pi_{33}^{\mu\nu(b)}(k)$ is the self-energy of the ghost tadpole diagram (b), being given by,

$$\begin{aligned} i\Pi_{33}^{\mu\nu(b)}(k) &= 2g^2 \delta^{aa'} \delta^{\mu\nu} \int \frac{d^d p}{(2^\pi)^d} \left[\frac{1}{p^2+S^2} \delta^{aa'} \right] \\ &= 4\delta^{\mu\nu} \int \frac{d^d p}{(2^\pi)^d} \left[\frac{1}{p^2+S^2} \right], \end{aligned} \tag{5.23}$$

  where 1 is the symmetry factor.

- $i\Pi_{33}^{\mu\nu(c)}(k)$ is the self-energy of the gluon sunrise diagram (c), being given by,

$$
\begin{aligned}
i\Pi_{33}^{\mu\nu(c)}(k) &= -g^2 f^{ab3} f^{a'b'3} \int \frac{d^dp}{(2^\pi)^d} \left[\frac{\delta^{aa'}}{p^2+M^2} \left(\delta^{\sigma\sigma'} - \frac{(1-\alpha)p^\sigma p^{\sigma'}}{p^2+\alpha M^2} \right) \times \right. \\
&\times \left(\delta^{\sigma\tau}(p-(-k-p))^\mu + \delta^{\mu\tau}((-k-p)-k)^\sigma + \delta^{\mu\sigma}(k-p)^\tau + \right. \\
&- \left. \frac{1}{\alpha}((-k-p)^\tau \delta^{\sigma\mu} - p^\sigma \delta^{\tau\mu}) \right) \left(\delta^{\sigma'\tau'}(-p-(k+p))^\nu + \right. \\
&+ \delta^{\nu\tau'}((k+p)-(-k))^{\sigma'} + \delta^{\nu\sigma'}(-k-(-p))^{\tau'} + \\
&- \left. \frac{1}{\alpha}((k+p)^{\tau'} \delta^{\sigma'\nu} - (-p)^{\sigma'} \delta^{\tau'\nu}) \right) \times \\
&\times \left. \frac{\delta^{bb'}}{(k+p)^2+M^2} \left(\delta^{\tau\tau'} - \frac{(1-\alpha)(k+p)^\tau (k+p)^{\tau'}}{(k+p)^2+\alpha M^2} \right) \right], \qquad (5.24)
\end{aligned}
$$

where 1 is the symmetry factor.

- $i\Pi_{33}^{\mu\nu(d)}(k)$ is the self-energy of the gluon tadpole diagram (d), being given by,

$$
\begin{aligned}
i\Pi_{33}^{\mu\nu(d)}(k) &= g^2 \left[f^{ac'3} f^{ac3} \left(\delta^{\mu\nu}\delta^{\tau\tau'} - \delta^{\tau\nu}\delta^{\mu\tau'} + \frac{1}{\alpha}\delta^{\tau'\nu}\delta^{\tau\mu} \right) + \right. \\
&+ \left. f^{ac3} f^{ac'3} \left(\delta^{\mu\nu}\delta^{\tau'\tau} - \delta^{\tau'\nu}\delta^{\mu\tau} + \frac{1}{\alpha}\delta^{\tau\nu}\delta^{\tau'\mu} \right) \right] \times \\
&\times \int \frac{d^dp}{(2\pi)^d} \left[\frac{1}{p^2+M^2} \left(\delta^{\tau\tau'} - \frac{(1-\alpha)p^\tau p^{\tau'}}{p^2+\alpha M^2} \right) \delta^{cc'} \right] \\
&= 2g^2 f^{ac3} f^{ac3} \delta^{\mu\nu} \int \frac{d^dp}{(2\pi)^d} \left[- \frac{(1-\alpha)\left(1-\frac{1}{d}+\frac{1}{d\alpha}\right)}{(p^2+M^2)(p^2+\alpha M^2)} + \right. \\
&+ \left. \frac{\left(d-1+(\frac{1}{\alpha})\right)}{p^2+M^2} \right], \qquad (5.25)
\end{aligned}
$$

where 1 is the symmetry factor.

After solving the integrals of eqs. (5.22), (5.23), (5.24) and (5.25) in order to obtain the solution of Abelian gluon self-energy of eq. (5.21), we contracted it with the external legs of these 1-loop diagrams and multiplyied the result by $\frac{1}{D-1}P_{\mu\nu}$ in order to take only the transverse sector and, changing $k \leftrightarrow p$, we found,

$$
\begin{aligned}
\Pi_{diag}(p^2) &= \frac{3g^2}{8(D-1)M^4\left(m^2+p^2\right)^2}\Bigg[2p^2\eta\left(M^2,\alpha M^2\right)\Bigg(2M^2p^2(\alpha-2D+3)+ \\
&+ (\alpha-1)^2M^4+p^4\Bigg)-\left(4M^2+p^2\right)\eta\left(M^2,M^2\right)\times \\
&\times \left(4(D-1)M^4+4(3-2D)M^2p^2+p^4\right)+32M^4S^2\eta\left(S^2,S^2\right)+ \\
&+ 8M^4p^2\eta\left(S^2,S^2\right)-p^6\eta\left(\alpha M^2,\alpha M^2\right)+4M^2p^4\eta\left(\alpha M^2,\alpha M^2\right)+ \\
&- 4\alpha M^2p^4\eta\left(\alpha M^2,\alpha M^2\right)-16\alpha M^6\eta\left(\alpha M^2,\alpha M^2\right)+ \\
&- 4M^4p^2\eta\left(\alpha M^2,\alpha M^2\right)+16\alpha M^4p^2\eta\left(\alpha M^2,\alpha M^2\right)-8D^2M^4\chi\left(M^2\right)+ \\
&- 16DM^4\chi\left(\sigma^2\right)+2M^2p^2(\alpha+4D-9)\chi\left(\alpha M^2\right)-8DM^2p^2\chi\left(M^2\right)+ \\
&- 8(D-2)M^4\chi\left(\alpha M^2\right)+24DM^4\chi\left(M^2\right)-2\alpha M^2p^2\chi\left(M^2\right)+ \\
&+ 18M^2p^2\chi\left(M^2\right)-16M^4\chi\left(M^2\right)\Bigg],
\end{aligned}
\tag{5.26}
$$

where we used the relations of eqs. (5.19) and (5.20) in order to write the solution in a compactified fashion. Then taking the gauge parameter $\alpha \to 0$ and the ghost effective mass $S^2 \to 0$ we obtain,

$$\begin{aligned}
\Pi_{diag}(p^2) &= \frac{3g^2}{8(D-1)M^4\left(m^2+p^2\right)^2}\Bigg[2p^2\eta\left(M^2,0\right)\Bigg(2(3-2D)M^2p^2 + \\
&+ M^4+p^4\Bigg) + \left(-4M^2-p^2\right)\eta\left(M^2,M^2\right)\Bigg(4(D-1)M^4 + \\
&+ 4(3-2D)M^2p^2+p^4\Bigg) + 4M^4p^2\eta(0,0) + 4M^2p^4\eta(0,0) + \\
&+ p^6(-\eta(0,0)) - 8D^2M^4\chi\left(M^2\right) - 8DM^2p^2\chi\left(M^2\right) + \\
&+ 24DM^4\chi\left(M^2\right) + 18M^2p^2\chi\left(M^2\right) - 16M^4\chi\left(M^2\right)\Bigg]. \qquad (5.27)
\end{aligned}$$

This is the total correction at 1-loop of the Abelian gluon two-point function. So our new 1-loop corrected Abelian gluon propagator will be,

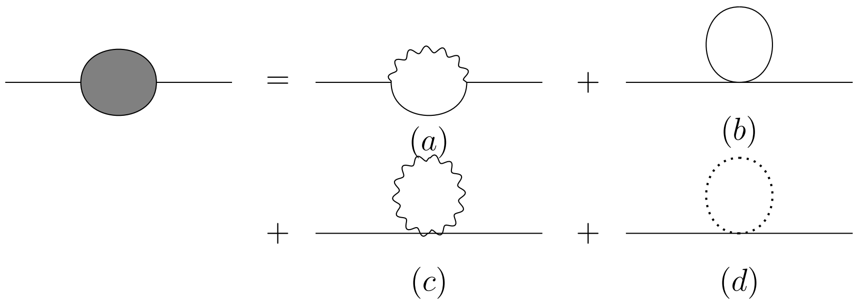
$$G_{diag}(p^2) = \frac{1}{p^2+m^2-\Pi_{diag}(p^2)}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\delta^{33}, \qquad (5.28)$$

which means that the form factor that we will analyze will be,

$$D_{diag}(p^2,m^2) = \frac{1}{p^2+m^2-\Pi_{diag}(p^2)}. \qquad (5.29)$$

5.3.2 Non-Abelian gluon self-energy

As well as in the previous subsection, let's now write down the total self-energy $i\Pi^{\alpha\nu}_{ab}(k)$ of the non-Abelian gluon two-point function, which is given by the sum of the diagrams below.



In other words,

$$i\Pi^{\mu\nu}_{ab}(k) = i\Pi^{\mu\nu(a)}_{ab}(k) + i\Pi^{\mu\nu(b)}_{ab}(k) + i\Pi^{\mu\nu(c)}_{ab}(k) + i\Pi^{\mu\nu(d)}_{ab}(k) \tag{5.30}$$

where,

- $i\Pi^{\mu\nu(a)}_{ab}(k)$ is the gluon sunrise (a) self-energy, being given by,

$$\begin{aligned}
i\Pi^{\mu\nu(a)}_{ab}(k) &= -\frac{1}{2} g^2 f^{ac3} f^{bd3} \int \frac{d^d p}{(2\pi)^d} \Bigg(\Bigg[\delta^{\mu\tau}(k - (-k-p))^{\sigma} + \\
&+ \delta^{\sigma\tau}(-k-p-p)^{\mu} + \delta^{\sigma\mu}(p-k)^{\tau} + \\
&- \frac{1}{\alpha}((-k-p)^{\tau}\delta^{\mu\sigma} - k^{\mu}\delta^{\tau\sigma}) \Bigg] \frac{\delta^{cd}}{(k+p)^2 + M^2} \times \\
&\times \left(\delta^{\tau\tau'} - \frac{(1-\alpha)(k+p)^{\tau}(k+p)^{\tau'}}{(k+p)^2 + \alpha M^2} \right) \frac{\delta^{33}}{p^2 + m^2} \times \\
&\times \left(\delta^{\sigma\sigma'} - \frac{p^{\sigma} p^{\sigma'}}{p^2} \right) \Bigg[\delta^{\tau'\nu}(k + p - (-k))^{\sigma'} + \delta^{\sigma'\nu}(-k - (-p))^{\tau'} + \\
&+ \delta^{\sigma'\tau'}(-p - (k+p))^{\nu} - \frac{1}{\alpha}(-k^{\nu}\delta^{\tau'\sigma'} - (k+p)^{\tau'}\delta^{\nu\sigma'}) \Bigg] \Bigg),
\end{aligned} \tag{5.31}$$

where $\frac{1}{2}$ is the symmetry factor.

- $i\Pi_{ab}^{\mu\nu(b)}(k)$ is the non-Abelian gluon tadpole (b) self-energy, being given by,

$$\begin{aligned}
i\Pi_{ab}^{\mu\nu(b)}(k) &= \frac{g^2}{2}\Bigg(f^{c'c3}f^{ba3}\left(\delta^{\nu\tau'}\delta^{\mu\tau} - \delta^{\mu\tau'}\delta^{\nu\tau}\right) + f^{ac'3}f^{cb3}\Big(\delta^{\tau\mu}\delta^{\nu\tau'} + \\
&- \delta^{\tau\tau'}\delta^{\nu\mu}\Big) + f^{c'b3}f^{ac3}\left(\delta^{\mu\tau'}\delta^{\tau\nu} - \delta^{\tau\tau'}\delta^{\mu\nu}\right)\Bigg) \times \\
&\times \int \frac{d^dp}{(2\pi)^d}\left[\frac{\delta^{cc'}}{p^2+M^2}\left(\delta^{\tau\tau'} - \frac{(1-\alpha)p^\tau p^{\tau'}}{p^2+\alpha M^2}\right)\right] \\
&= g^2 f^{ac3}f^{cb3}\int \frac{d^dp}{(2\pi)^d}\Bigg[-\frac{(1-\alpha)(p^\mu p^\nu - p^2\delta^{\mu\nu})}{(p^2+M^2)(p^2+\alpha M^2)} + \\
&+ \frac{(1-d)\delta^{\mu\nu}}{p^2+M^2}\Bigg], \qquad (5.32)
\end{aligned}$$

where $\frac{1}{2}$ is the symmetry factor.

- $i\Pi_{ab}^{\mu\nu(c)}(k)$ is the Abelian gluon tadpole (c) self-energy, being given by,

$$\begin{aligned}
i\Pi_{ab}^{\mu\nu(c)}(k) &= g^2\Bigg[f^{db3}f^{da3}\left(\delta^{\tau\tau'}\delta^{\mu\nu} - \delta^{\mu\tau'}\delta^{\tau\nu} + \frac{1}{\alpha}\delta^{\nu\tau'}\delta^{\mu\tau}\right) + \\
&+ f^{da3}f^{db3}\left(\delta^{\tau\tau'}\delta^{\nu\mu} - \delta^{\nu\tau'}\delta^{\tau\mu} + \frac{1}{\alpha}\delta^{\mu\tau'}\delta^{\nu\tau}\right)\Bigg] \times \\
&\times \int \frac{d^dp}{(2\pi)^d}\left[\frac{1}{p^2+m^2}\left(\delta^{\tau\tau'} - \frac{p^\tau p^{\tau'}}{p^2}\right)\delta^{33}\right] \\
&= 2g^2\delta^{ab}\delta^{\mu\nu}\left(d-2+\frac{1}{\alpha}+\frac{1}{d}-\frac{1}{d\alpha}\right)\int \frac{d^dp}{(2\pi)^d}\left[\frac{1}{p^2+m^2}\right], \qquad (5.33)
\end{aligned}$$

where 1 is the symmetry factor and $f^{db3}f^{da3} = f^{bd3}f^{ad3} = f^{3bd}f^{3ad} = \delta^{ab}$.

- $i\Pi^{\mu\nu(d)}_{ab}(k)$ is the ghost tadpole (d) self-energy, being given by,

$$i\Pi^{\mu\nu}_{ab}(k) = g^2\delta^{\mu\nu}\left(f^{c'a3}f^{cb3}+f^{c'b3}f^{ca3}\right)\int\frac{d^dp}{(2\pi)^d}\left[\frac{\delta^{cc'}}{p^2+S^2}\right], \quad (5.34)$$

where 1 is the symmetry factor.

Like in the Abelian case, after solving the integrals of eqs. (5.31), (5.32), (5.33) and (5.34) in order to obtain the solution of non-Abelian gluon self-energy of eq. (5.30), we contracted it with its external legs, then multiplied the result by $\frac{1}{D-1}P_{\mu\nu}$ in order to consider the transverse sector only and, changing $k\leftrightarrow p$, we found,

$$\begin{aligned}
\Pi^T_{off}(p^2) &= \frac{g^2\delta^{ab}}{4(D-1)p^2m^2M^2\left(p^2+M^2\right)^2}\Bigg[\left(p^2+m^2+M^2\right)^2\eta\left(m^2,\alpha M^2\right)\times\\
&\times\left(-2m^2\left((2D-3)p^2+\alpha M^2\right)+\left(p^2+\alpha M^2\right)^2+m^4\right)+\\
&-\left(-2m^2\left(M^2-p^2\right)+\left(p^2+M^2\right)^2+m^4\right)\eta\left(M^2,m^2\right)\times\\
&\times\left(2(2D-3)m^2\left(M^2-p^2\right)+2(3-2D)p^2M^2+p^4+m^4+M^4\right)+\\
&+\left(p^2+M^2\right)^2\eta\left(M^2,0\right)\left(2(3-2D)p^2M^2+p^4+M^4\right)-\left(p^2+M^2\right)^2\times\\
&\times\left(p^2+\alpha M^2\right)^2\eta\left(\alpha M^2,0\right)-M^2\chi\left(m^2\right)\times\\
&\times\Big(2m^2\left(p^2\left(\alpha+2D^2-8D+7\right)+M^2(\alpha-2D+3)\right)+\left(p^2+M^2\right)\times\\
&\times\left(p^2(\alpha+4D-5)+(\alpha-1)M^2\right)+m^4(\alpha+4D-9)\Big)+\\
&+m^2\Big(\chi\left(M^2\right)\left(-2\left(2D^2-8D+7\right)p^2M^2+(7-4D)p^4+\right.\\
&+\left.(4D-7)m^2\left(M^2-p^2\right)+(7-4D)M^4+m^4\right)+\\
&+8(D-1)p^2M^2\chi\left(S^2\right)\Big)+m^2\chi\left(\alpha M^2\right)\Big(-\left((7-4D)p^4+\right.\\
&+m^2\left((7-4D)p^2-(\alpha-2)M^2\right)-2p^2M^2(\alpha+4D-6)+m^4+\\
&+\left.(1-2\alpha)M^4\right)\Big)\Bigg],
\end{aligned} \quad (5.35)$$

where we also used the relations of eqs. (5.19) and (5.20) in order to write it in a simpler form. Then taking the gauge parameter $\alpha \to 0$ and the ghost effective mass $S^2 \to 0$ we obtain,

$$
\begin{aligned}
\Pi^T_{off}(p^2) &= \frac{g^2\delta^{mn}}{4(D-1)k^2m^2M^2\left(k^2+M^2\right)^2}\Bigg[\eta\left(m^2,0\right)\Bigg(-2(2D-3)k^2m^2 + \\
&+ k^4+m^4\Bigg)\left(k^2+m^2+M^2\right)^2 - \Bigg(-2m^2\left(M^2-k^2\right) + \\
&+ \left(k^2+M^2\right)^2+m^4\Bigg)\eta\left(M^2,m^2\right)\Bigg(2(2D-3)m^2\left(M^2-k^2\right) + \\
&+ 2(3-2D)k^2M^2+k^4+m^4+M^4\Bigg) + \left(k^2+M^2\right)^2\eta\left(M^2,0\right)\times \\
&\times \left(2(3-2D)k^2M^2+k^4+M^4\right) - k^4\eta(0,0)\left(k^2+M^2\right)^2 + \\
&- M^2\chi\left(m^2\right)\Bigg(2m^2\left(\left(2D^2-8D+7\right)k^2+(3-2D)M^2\right) + \\
&+ \left(k^2+M^2\right)\left((4D-5)k^2-M^2\right)+(4D-9)m^4\Bigg) + m^2\chi\left(M^2\right)\times \\
&\times \Bigg(-2\left(2D^2-8D+7\right)k^2M^2+(7-4D)k^4+(4D-7)m^2\times \\
&\times \left(M^2-k^2\right)+(7-4D)M^4+m^4\Bigg)\Bigg],
\end{aligned}
\tag{5.36}
$$

which is the total correction at 1-loop of the non-Abelian gluon two-point function. This means that the 1-loop corrected non-Abelian transverse gluon propagator will be given by,

$$
G^T_{off}(p^2) = \frac{1}{p^2+M^2-\Pi_{off}(p^2)}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\delta^{ab}, \tag{5.37}
$$

which means that the form factor that we will analyze will be,

$$
D^T_{off}(p^2,m^2) = \frac{1}{p^2+M^2-\Pi_{off}(p^2)}. \tag{5.38}
$$

In a similar way, the longitudinal sector of the non-Abelian gluon self-energy (5.30) can be obtained on Mathematica and it is given by,

$$\begin{aligned}\Pi^L_{off}(p^2) &= \frac{-g^2}{4m^2M^2p^6}\Bigg[-p^4\eta\left(M^2,m^2\right)\Bigg(2m^2\left((2D-3)M^2+p^2\right)+m^4+\\ &+ (M^2+p^2)^2\Bigg)+p^4\left(m^4+2m^2p^2+p^4\right)\eta\left(m^2,0\right)+\\ &+ p^4(M^2+p^2)^2\eta\left(M^2,0\right)-p^8\eta\left(0,0\right)+M^2p^4\chi(m^2)+\\ &+ m^2p^4\chi(M^2)\Bigg]. \qquad (5.39)\end{aligned}$$

From it we obtain the 1-loop corrected longitudinal non-Abelian gluon propagator, which will be given by,

$$G^L_{off}(p^2) = \frac{\alpha}{p^2+\alpha M^2}L_{\mu\nu}(p^2)\delta^{ab}. \qquad (5.40)$$

As we can see, the longitudinal term $G^L_{off}(p^2)$ is proportional to the gauge parameter α, meaning that in the limit of $\alpha \to 0$ it becomes identically null. Because of this in the next section we will focus our analysis only to the transverse sector of gluon two-point functions. However, we might state that this does not mean necessarily that the longitudinal sector of the Abelian and non-Abelian gluon two-point functions might not be affected by the addition of the effective masses m and M, since at higher orders in the loop expansion a longitudinal contribution can still appear.

5.4 Analysis of the Abelian and non-Abelian sectors

After having calculated both the self-energies of the Abelian and non-Abelian gluon sectors of the theory, we are finally able to fit our results with the ones obtained in the Bornyakov's work [4]. In its work, Bornyakov calculated on the lattice the gluon propagators in the Maximal abelian gauge and plotted their form factors and dressing functions, as we can see on figure 5.1. After that he compared the data obtained with the form factors and dressing functions of the most prominent massive models at the time to describe the infrared regime of the Yang-Mills theory. To do so, he fitted the following functions,

$$fit_1(Z,m,a) = \frac{Zm^{2a}}{(m^2+p^2)^{a+1}}, \tag{5.41}$$

$$fit_2(Z,m,a) = \frac{Zm^{2a}}{m^{2(a+1)}+p^{2(a+1)}}, \tag{5.42}$$

$$fit_Y(Z,m,a) = \frac{Z}{m^2+p^2}, \tag{5.43}$$

$$fit_{Y2}(Z,m,k) = \frac{Z}{\frac{\kappa p^4}{m^2}+m^2+p^2}, \tag{5.44}$$

$$fit_G(Z,m,p) = \frac{p^2 Z}{m^4+p^4}, \tag{5.45}$$

with the graphics of figure 5.1. Here, the functions (5.43) and (5.44) are Yukawa type form factors and (5.45) is a Gribov-like one.

It was found through fits that in the limit of low momenta $p \to 0$, the Gribov form factor did not provide a good fit to the data. In every other instance, however, it was found that these models not only provided a good qualitative match to the data, but also replicated propagators that showed the Abelian predominance in the theory. The outcomes of the fitted parameters m, α, κ, and Z can

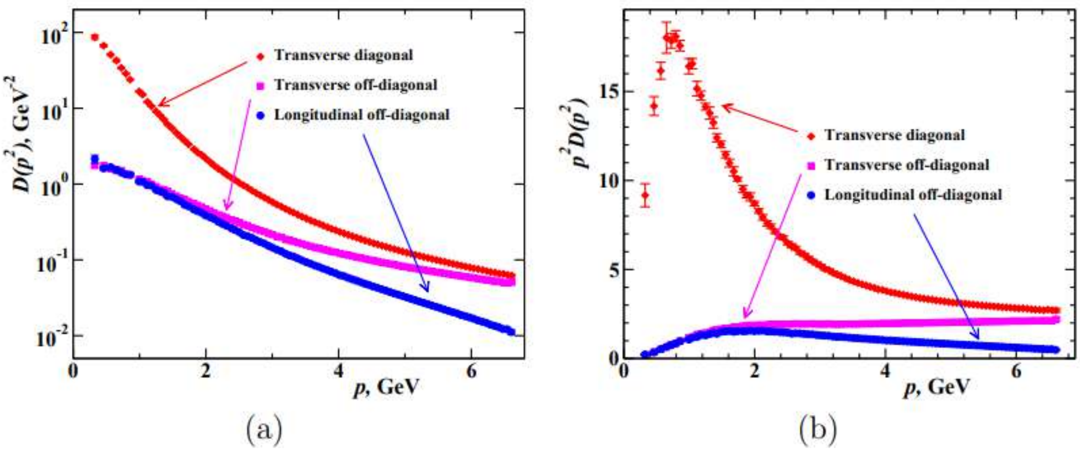


Figure 5.1: (a) The form factors $D(p^2)$, and (b) the gluon dressing functions $p^2D(p^2)$ in terms of the momentum p. This figures were obtained on lattice in the Bornyakov's work [4].

be seen in figure 5.2. There it can be seen that the non-Abelian gluon masses are at least two times bigger than the Abelian ones which is an evidence of Abelian dominance in the infrared regime.

However, between all the fittings, the function (5.41) was the one that better reproduced the qualitative and numerical behaviour of the lattice propagators. With this in mind, we fitted our own form factors (5.29) and (5.38) with the function (5.41).

To be more precise, what we did was to reproduce the fitting of the function (5.41) obtained by Bornyakov for the Abelian gluon propagator. After that, we fitted our Abelian form factor (5.29) with (5.41) in order to determine all of our free parameters, as we can see on figure 5.3. Doing so, we obtained that $m = 0.308GeV$, $M = 1.37GeV$, $\mu = 2.82$ and $g = 2.18$, which are respectively the Abelian effective mass, the non-Abelian effective mass, the renormalizarion constant that comes from the $\bar{M}S$ renormalization scheme, and the coupling constant. We can see here that the non-Abelian mass is much bigger than Abelian one, which is a good evidendce of the Abelian dominance in the infrared regime.

fit	m, GeV	α or κ	Z	$p_{\max}$, GeV	χ^2/dof
Transverse diagonal					
fit 1	0.73(2)	0.92(3)	16.9(4)	1.7	0.8
fit 2	0.58(2)	0.49(5)	8.5(2)	1.0	0.4
Gribov fit	0.33(1)	-	4.58(5)	0.9	0.9
Yukawa 2 fit	0.50(2)	0.19(3)	8.3(3)	1.7	0.9
Transverse off-diagonal					
fit 1	1.6(2)	0.6(2)	1.3(2)	1.7	1.0
fit 2	1.26(4)	0.19(4)	0.73(2)	1.7	1.0
Yukawa fit	1.08(2)	0	0.63(1)	1.7	1.5
Yukawa 2 fit	1.29(6)	0.15(5)	0.81(5)	1.7	1.0
Longitudinal off-diagonal					
fit 1	1.4(2)	0.5(3)	1.0(3)	1.7	1.1
fit 2	1.14(6)	0.19(6)	0.63(3)	1.7	1.1
Yukawa fit	0.96(3)	0	0.52(1)	1.7	1.3
Yukawa 2 fit	1.14(8)	0.12(7)	0.66(6)	1.7	1.1

Figure 5.2: Graphic of the dressing function $p^2D(p^2)$ vs p^2. In blue we have the transverse Abelian dressing function while in orange we have the fitting of the function in the Bornyakov's work [4].

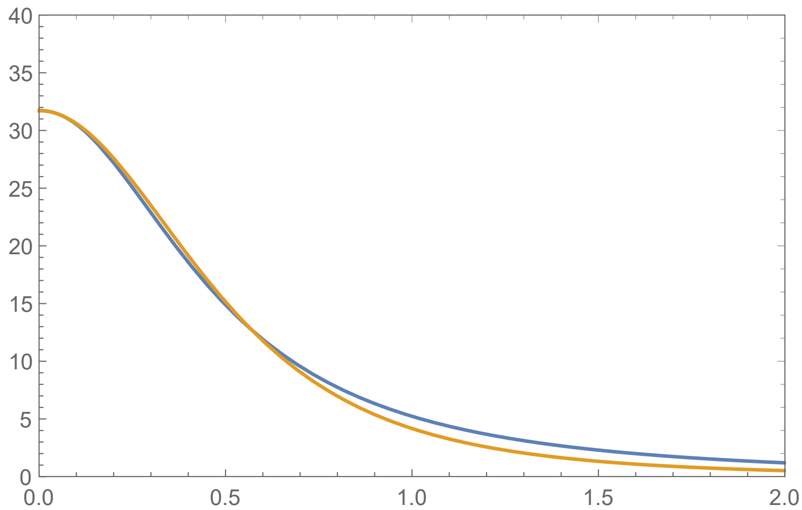


Figure 5.3: Graphic of the form factor $D(p^2)$ vs p^2. In blue we have the transverse Abelian form factor while in orange we have the fitting of the function (5.41) that was fitted with lattice in the Bornyakov's work [4].

After that, we substituted these values on the non-Abelian gluon form factor (5.38) in order to compare its behaviour with the abelian one, as we can see on figure 5.4. Thankfully, what we obtained was a really nice graphic in which it can be observed the dominant behaviour of the transverse Abelian form factor with respect to the non-Abelian one, which is the most important conclusion of this chapter.

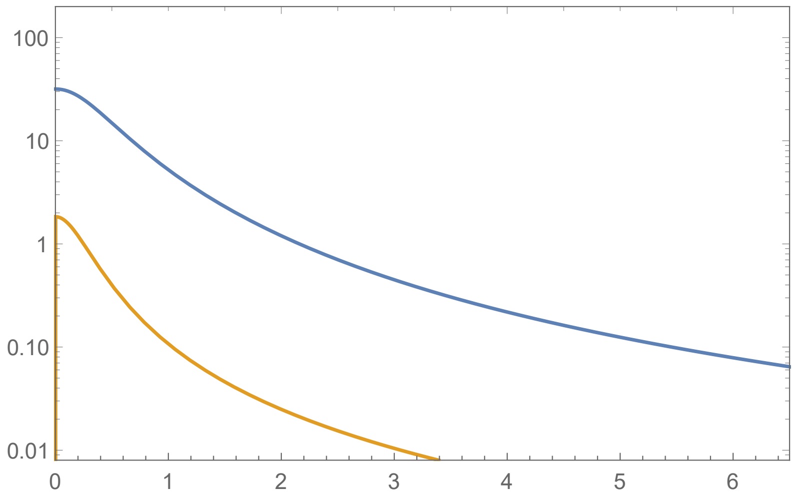


Figure 5.4: Graphic of the form factor $D(p^2)$ vs p^2. In blue we have the transverse Abelian form factor while in orange we have the transverse non-Abelian one.

Chapter 6

Conclusions

In this thesis we explore different methods to obtain information about all the nuances regarding the Yang-Mills theory in the infrared regime. In particular, we were interested in two main features: the Abelian dominance conjecture and the gluonic phase transition between confined and de-confined regimes.

To do so, we followed 3 different paths.

- The first path was to investigate the Yang-Mills-Chern-Simons theory, since it is an effective 3-dim field theory to describe the 3-dim Yang-Mills-Higgs model, which in its turn is an effective field theory to describe the 4-dim Yang-Mills theory at high temperatures. In such a way, it can be seen as a toy-model to study the Yang-Mills at high temperatures, where the Chern-Simons mass contains all the thermal information of the model.

  By adding the horizon function in order to eliminate all the infinitesimal Gribov copies of the theory, it adds a novel massive term called Gribov parameter which couples to the gluon fields. Such parameter is determined via gap eq. and modifies the pole structure of the gluon propagator generating degrees of freedom with complex masses. This is uderstood as an indication of confinement since such degrees of freedom are necessarily out of the physical spectrum and, more than that, they lead to a positivity iolation of the spectral density, which are both conditions that need to be fulfilled in order to consider a particle to be confined in the infrared regime.

Besides that, due to a quantization rule imposed in order to grant the gauge invariance of the Chern-Simons action, we found out that by restrincting the domain of the path integral to the Gribov region by adding the horizon function, we eliminated not only the infinitesimal Gribov copies but also all the finite ones, giving rise to a path integral completely free from Gribov copies.

Another important result that was obtained was the fact that the topological Chern-Simons mass modifies the gap eq. that determined the Gribov parameter, which is something that was not obtained in previous works [81] in Landau gauge, meaning that these are novel findings.

This work led us to publish two articles [71; 74], one where we analyzed the Yang-Mills-Chern-Simons theory in the maximal Abelian gauge and the other we analyzed it whithin the linear covariant ones.

In the linear covariant gauges, we obtained results that were in good qualitative agreement with the ones obtained in [84; 85] for Yang-Mills-Higgs theory. This study led us to obtain a regime with two masses, one positive and one negative for the gluon propagator, and when the weak coupling condition is met, the standard particle spectrum is obtained. The Gribov gap equation in pure Yang-Mills theory at finite temperatures produced a phase diagram similar to previous studies [84; 85], with temperature substituting for the Higgs vacuum expectation value.

Yet in the maximal Abelian gauge we were able to analyze the Abelian and non-Abelian sectors of the theory separately thanks to the Cartan depomposition. After calculating the propagators of the theory we saw, as it was already expected, that the Gribov parameter enters only into the poles of the Abelian propagator. This means that the model is distinct from standard Yang-Mills theories linked to Higgs fields, as the off-diagonal components are always deconfined, and the Abelian component can have physical excitations based on the value of the topological mass. The shift from confinement to deconfinement is influenced solely by the existence of the topological mass and is not dependent on any additional fields.

- In the second path we investigated the Refined Gribov-Zwanziger model at finite temperature to understand the effects of the Gribov parameter and the 2-dim condensate masses on the pole structure of propagators and phase transitions of the theory. We used a semi-classical approach in which we introduced an effective massive Yang-Mills model in which the effective massive terms incorporated all the thermal effects. We identified two critical temperatures for phase transitions and we showed that the masses of the condensates decrease the temperatures of phase transitions. The intermediate phase of partial confinement is an essential finding as it agrees with other works [81; 104; 105; 106; 107] on the quark-gluon plasma's transition behavior. Then we concluded by discussing the complexity of extending such method to 1-loop order the results and our plans for future research, including calculating condensate masses at finite temperatures and analyzing the longitudinal and transverse masses in terms of temperature, since this would increase the precision of our results.

- In the third path we built an effective massive Yang-Mills model in order to study the infrared regime of the Yang-Mills theory within the maximal Abelian gauge. Basically, we added two massive terms inside which all the non-perturbative information of theory is contained. After that, we calculated at 1-loop order the gluon self energy of both the Abelian and non-Abelian sectors of the model. Then we used them to write the Abelian and non-Abelian gluon propagators with their masses corrected at 1-loop. Finally we fitted our Abelian and non-Abelian form factors with functions that where previosly fitted with lattice results on [4]. In such a way, we could determine the value of the Abelian and non-Abelian effective masses that we introduced at the beginning of the work into the Yang-Mills action. We concluded then that not only our model reproduces very well the lattice resimulations as also reveals to us that the non-Abelian mass is considerably bigger that the Abelian one, which is a really important result, since it is a good evidence of Abelian dominance in the theory.

Said that, we must emphasize that even though these studies led to really nice results that were in good qualitative, and in some cases even numerically, agreement with lattice, we still intend to improve the precision of our results in future works. Between them we have:

- The calculation the gluon self energy of the Refined Gribov-Zwanziger model at 1-loop, in order to be able to plot the form factor of the gluon propagator with its first order corrections and compare it with lattice. This is a work that is already being done in the linear covariant gauges and might to be sent to publish as sooner as possible.
- The calculation of the gluon self energy of the Yang-Mills-Chern-Simons theory within the Refined Gribov-Zwanziger framework in linear covariant gauges at 1-loop. This is also a work that was already started.
- The calculation of the masses of the 2-dim condensates at finite temperatures. This would be interesting since it would allow us to have a better understanding on how such masses, together with the Gribov parameter

affect the phase transition of the Yang-Mills theory at 1-loop, since this would give us much more precise results for the effective thermal masses calculated in this thesis.

- The study of the $SU(N)$ massive Yang-Mills theory in the maximal Abelian gauge. This work would be interesting since it would extend to the $SU(N)$ the study of this thesis, which was done only in the $SU(2)$.

- The calculation of the gluon and quark two-point functions at 1-loop within the Refined Gribov-Zwanziger framework. This work would be interesting since, as already stated in previows works [108; 109], the Gribov-Zwanziger model allows us to add a novel term into the action that couples the Faddeev-Popov operator to the quark sector leading to the arising of a Gribov-like mass into the poles of the quark propagator. Hence, it would be interesting to analyze how this modifies both the gluon and quark two-point funcions behaviour at 1-loop.

References

[1] A. D. P. Junior, "Exploring new horizons of the gribov problem in yang-mills theories," xi, 13

[2] D. Fiorentini, *Non-perturbative exact nilpotent BRST symmetry for the Gribov-Zwanziger action.* PhD thesis, 05 2018. xi, 29

[3] A. D. Pereira, *Exploring new horizons of the Gribov problem in Yang-Mills theories.* PhD thesis, Niteroi, Fluminense U., 2016. xi, 44

[4] V. Bornyakov, M. Chernodub, F. Gubarev, S. Morozov, and M. Polikarpov, "Abelian dominance and gluon propagators in the maximally abelian gauge of su (2) lattice gauge theory," *Physics Letters B*, vol. 559, no. 3-4, pp. 214–222, 2003. xiii, 22, 110, 111, 112, 113, 127, 128, 129, 134

[5] C. N. Yang and R. L. Mills, "Conservation of Isotopic Spin and Isotopic Gauge Invariance," *Physical Review*, vol. 96, pp. 191–195, Oct. 1954. 1, 6

[6] V. Rubakov, "Classical theory of gauge fields," in *Classical Theory of Gauge Fields*, Princeton University Press, 2009. 1

[7] M. Atiyah, "Geometry of yang-mills fields," *Lezioni Fermiane, Accademia Nazionale dei Lincei Scuola Normale Superione, Pisa*, 1979. 1

[8] G. V. Dunne, "Aspects of chern-simons theory," 1999. 2, 52, 53, 57, 58

[9] T. Appelquist and R. D. Pisarski, "High-temperature yang-mills theories and three-dimensional quantum chromodynamics," *Phys. Rev. D*, vol. 23, pp. 2305–2317, May 1981. 3, 52, 58

[10] D. J. Gross and F. Wilczek, "Asymptotically free gauge theories. i," *Phys. Rev. D*, vol. 8, pp. 3633–3652, Nov 1973. 3, 6

[11] D. J. Gross and F. Wilczek, "Asymptotically free gauge theories. ii," *Phys. Rev. D*, vol. 9, pp. 980–993, Feb 1974. 3, 6

[12] D. J. Gross and F. Wilczek, "Ultraviolet behavior of non-abelian gauge theories," *Phys. Rev. Lett.*, vol. 30, pp. 1343–1346, Jun 1973. 3

[13] F. Canfora, A. Gomez, S. P. Sorella, and D. Vercauteren, "Study of yang–mills–chern–simons theory in presence of the gribov horizon," *Annals of Physics*, vol. 345, pp. 166–177, 2014. 3, 54, 56, 57, 58, 60, 61, 69

[14] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, and H. Verschelde, "Refinement of the gribov-zwanziger approach in the landau gauge: Infrared propagators in harmony with the lattice results," *Physical Review D*, vol. 78, no. 6, p. 065047, 2008. 6, 20, 47

[15] D. Dudal, S. Sorella, and N. Vandersickel, "Dynamical origin of the refinement of the gribov-zwanziger theory," *Physical Review D*, vol. 84, no. 6, p. 065039, 2011. 6, 47

[16] L. D. Landau and E. M. Lifshitz, "Classical field theory," *Course of Theoretical Physics*, vol. 2, 1975. 9

[17] A. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie," *Annalen der Physik*, vol. 354, pp. 769–822, 1916. 11

[18] L. M. Brown, *Feynman's Thesis — A New Approach to Quantum Theory.* WORLD SCIENTIFIC, 2005. 11

[19] L. H. Ryder, *Quantum field theory.* Cambridge university press, 1996. 11, 13, 14, 16, 17

[20] S. Weinberg, *The quantum theory of fields*, vol. 2. Cambridge university press, 1995. 11, 17, 36

[21] M. E. Peskin, *An introduction to quantum field theory.* CRC Press, 2018. 11

[22] L. D. Faddeev and V. N. Popov, "Feynman diagrams for the Yang-Mills field," *Physics Letters B*, vol. 25, pp. 29–30, July 1967. 13, 17

[23] N. Nakanishi, "Covariant quantization of the electromagnetic field in the landau gauge," *Progress of Theoretical Physics*, vol. 35, no. 6, pp. 1111–1116, 1966. 17

[24] B. Lautrup, "Canonical quantum electrodynamics in covariant gauges," *MATEMATISK-FYSISKE MEDDELELSER UDGIVET AF DET KONGELIGE DANSKE VIDENSKABERNES SELSKAB*, vol. 35, no. 11, p. 3, 1967. 17

[25] R. F. Sobreiro, "Non-perturbative aspects of yang-mills theories. phd thesis," *arXiv preprint arXiv:0705.4107*, 2007. 17

[26] C. Becchi, A. Rouet, and R. Stora, "Renormalization of Gauge Theories," *Annals Phys.*, vol. 98, pp. 287–321, 1976. 18, 36

[27] C. M. Becchi and C. Imbimbo, "Becchi-rouet-stora-tyutin symmetry," *Scholarpedia*, vol. 3, no. 10, p. 7135, 2008. 18, 36

[28] I. Batalin, P. Lavrov, and I. Tyutin, "Covariant quantization of gauge theories in the framework of extended brst symmetry," *Journal of mathematical physics*, vol. 31, no. 6, pp. 1487–1493, 1990. 18, 36

[29] O. Piguet and S. P. Sorella, *Algebraic renormalization: Perturbative renormalization, symmetries and anomalies*, vol. 28. Springer Science & Business Media, 2008. 18, 36, 113

[30] A. W. Knapp, *Lie groups, Lie algebras, and cohomology*, vol. 34. Princeton University Press, 1988. 18

[31] C. S. Fischer, A. Maas, and J. M. Pawlowski, "On the infrared behavior of landau gauge yang–mills theory," *Annals of Physics*, vol. 324, no. 11, pp. 2408–2437, 2009. 20

[32] A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, and N. Strodthoff, "Landau gauge yang-mills correlation functions," *Physical Review D*, vol. 94, no. 5, p. 054005, 2016. 20

[33] A. Cucchieri, A. Maas, and T. Mendes, "Three-point vertices in landau-gauge yang-mills theory," *Physical Review D*, vol. 77, no. 9, p. 094510, 2008. 20

[34] C. S. Fischer and J. M. Pawlowski, "Uniqueness of infrared asymptotics in landau gauge yang-mills theory," *Physical Review D*, vol. 75, no. 2, p. 025012, 2007. 20

[35] R. Alkofer, M. Q. Huber, and K. Schwenzer, "Infrared singularities in landau gauge yang-mills theory," *Physical Review D*, vol. 81, no. 10, p. 105010, 2010. 20

[36] D. Zwanziger, "Covariant quantization of gauge fields without gribov ambiguity," *Nuclear Physics B*, vol. 192, no. 1, pp. 259–269, 1981. 20, 30, 32, 75

[37] A. Cucchieri and T. Mendes, "Landau-gauge propagators in yang-mills theories at β= 0: massive solution versus conformal scaling," *Physical Review D*, vol. 81, no. 1, p. 016005, 2010. 20

[38] A. G. Duarte, O. Oliveira, and P. J. Silva, "Lattice gluon and ghost propagators and the strong coupling in pure su (3) yang-mills theory: Finite lattice spacing and volume effects," *Physical Review D*, vol. 94, no. 1, p. 014502, 2016. 20, 36, 75

[39] V. N. Gribov, "Quantization of Nonabelian Gauge Theories," *Nucl. Phys.*, vol. B139, p. 1, 1978. [,1(1977)]. 20, 27, 29, 32, 75

[40] W. W. Tucker and J. D. Stack, "The maximal abelian gauge in su (3) lattice gauge theory," *Nuclear Physics B-Proceedings Supplements*, vol. 106, pp. 643–645, 2002. 22

[41] Z. F. Ezawa and A. Iwazaki, "Abelian dominance and quark confinement in yang-mills theories," *Phys. Rev. D*, vol. 25, pp. 2681–2689, May 1982. 22

[42] F. Gubarev, L. Stodolsky, and V. I. Zakharov, "On the significance of the vector potential squared," *Physical Review Letters*, vol. 86, no. 11, p. 2220, 2001. 23

[43] F. Gubarev and V. I. Zakharov, "Emerging phenomenology of," *Physics Letters B*, vol. 501, no. 1-2, pp. 28–36, 2001. 23

[44] D. Zwanziger, "Renormalizability of the critical limit of lattice gauge theory

by brs invariance," *Nuclear Physics B*, vol. 399, no. 2, pp. 477 – 513, 1993. 27, 29, 32, 44, 75

[45] R. F. Sobreiro *et al.*, "Aspectos não perturbativos das teorias de yang-mills.," 2007. 27, 29, 30, 31, 32

[46] N. Vandersickel, "A study of the gribov-zwanziger action: from propagators to glueballs," *arXiv preprint arXiv:1104.1315*, 2011. 27, 29, 30, 31, 32, 43, 65, 75

[47] M. Capri, "Aspectos não perturbativos das teorias de yang-mills no calibre abeliano maximal, phd thesis," 2009. 27, 29, 32, 36, 43, 113

[48] A. Pereira, R. Sobreiro, and S. Sorella, "Non-perturbative brst quantization of euclidean yang–mills theories in curci–ferrari gauges," *The European Physical Journal C*, vol. 76, no. 10, p. 528, 2016. 29

[49] D. Zwanziger, "Local and renormalizable action from the gribov horizon," *Nuclear Physics B*, vol. 323, no. 3, pp. 513–544, 1989. 30, 34, 75

[50] S. P. Sorella, "Remarks on the dynamical mass generation in confining Yang-Mills theories," *Brazilian Journal of Physics*, vol. 36, pp. 222 – 228, 03 2006. 30, 50

[51] G. Dell'Antonio and D. Zwanziger, "Every gauge orbit passes inside the gribov horizon," *Communications in mathematical physics*, vol. 138, no. 2, pp. 291–299, 1991. 30

[52] A. D. Pereira, "Exploring new horizons of the gribov problem in yang-mills theories," *arXiv preprint arXiv:1607.00365*, 2016. 31, 36

[53] M. E. Peskin and F. Halzen, "An introduction to quantum field theory, by michael e. peskin and daniel v. schroeder," 1995. 36, 87

[54] N. Maggiore and M. Schaden, "Landau gauge within the gribov horizon," *Physical Review D*, vol. 50, no. 10, p. 6616, 1994. 36

[55] M. Capri, D. Fiorentini, M. Guimaraes, B. Mintz, L. Palhares, S. Sorella, D. Dudal, I. Justo, A. Pereira, and R. Sobreiro, "Exact nilpotent nonperturbative brst symmetry for the gribov-zwanziger action in the linear covariant gauge," *Physical Review D*, vol. 92, no. 4, p. 045039, 2015. 36, 40, 67

[56] M. Capri, D. Fiorentini, A. Pereira, and S. Sorella, "Renormalizability of the refined gribov-zwanziger action in linear covariant gauges," *Physical Review D*, vol. 96, no. 5, p. 054022, 2017. 36, 40

[57] D. Zwanziger, "Quantization of gauge fields, classical gauge invariance and gluon confinement," *Nuclear Physics B*, vol. 345, no. 2-3, pp. 461–471, 1990. 36, 75

[58] M. Lavelle and D. McMullan, "Constituent quarks from qcd, phys," 1997. 36

[59] M. Capri, D. Dudal, D. Fiorentini, M. Guimaraes, I. Justo, A. Pereira, B. Mintz, L. Palhares, R. Sobreiro, and S. Sorella, "Local and brst-invariant yang-mills theory within the gribov horizon," *Physical Review D*, vol. 94, no. 2, p. 025035, 2016. 37

[60] F. Bruckmann, T. Heinzl, A. Wipf, and T. Tok, "Instantons and gribov copies in the maximally abelian gauge," *Nuclear Physics B*, vol. 584, no. 1-2, pp. 589–614, 2000. 42

[61] M. Capri, V. Lemes, R. Sobreiro, S. Sorella, and R. Thibes, "Study of the maximal abelian gauge in s u (2) euclidean yang-mills theory in the presence of the gribov horizon," *Physical Review D*, vol. 74, no. 10, p. 105007, 2006. 43, 67, 113

[62] M. A. L. Capri, S. P. Sorella, M. Guimaraes, and A. Gomez, "A study of the gribov problem for the maximal abelian gauge in su (n) yang-mills theories," 2009. 43

[63] M. A. L. Capri, A. Gomez, M. Guimaraes, V. E. R. Lemes, and S. P. Sorella, "Properties of gribov region and horizon function in the su (n) maximal abelian gauge," 2011. 43

[64] M. Capri, V. Lemes, R. Sobreiro, S. Sorella, and R. Thibes, "Local renormalizable gauge theories from nonlocal operators," *Annals of Physics*, vol. 323, no. 3, pp. 752–767, 2008. 44, 113

[65] D. Zwanziger, "Local and renormalizable action from the gribov horizon," *Nuclear Physics B*, vol. 323, pp. 513–544, 9 1989. 44

[66] A. Cucchieri and T. Mendes, "Constraints on the infrared behavior of the gluon propagator in yang-mills theories," *Physical review letters*, vol. 100, no. 24, p. 241601, 2008. 47, 49

[67] A. Cucchieri and T. Mendes, "Constraints on the infrared behavior of the ghost propagator in yang-mills theories," *Physical Review D*, vol. 78, no. 9, p. 094503, 2008. 47, 49

[68] A. Maas, "More on gribov copies and propagators in landau-gauge yang-mills theory," *Physical Review D*, vol. 79, no. 1, p. 014505, 2009. 47, 49

[69] D. Dudal, O. Oliveira, and P. J. Silva, "High precision statistical landau gauge lattice gluon propagator computation vs. the gribov–zwanziger approach," *Annals of Physics*, vol. 397, pp. 351–364, 2018. 47, 49

[70] D. Dudal, S. Sorella, N. Vandersickel, and H. Verschelde, "New features of the gluon and ghost propagator in the infrared region from the gribov-

zwanziger approach," *Physical Review D*, vol. 77, no. 7, p. 071501, 2008. 47

[71] L. C. Ferreira, D. R. Granado, I. F. Justo, and A. D. Pereira, "Infrared propagators of yang-mills-chern-simons theories in linear covariant gauges," *Physical Review D*, vol. 104, no. 4, p. 045007, 2021. 48, 49, 55, 89, 132

[72] D. Dudal, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella, and H. Verschelde, "An Analytic study of the off-diagonal mass generation for Yang-Mills theories in the maximal Abelian gauge," *Phys. Rev.*, vol. D70, p. 114038, 2004. 50

[73] D. Dudal, S. P. Sorella, and N. Vandersickel, "Dynamical origin of the refinement of the gribov-zwanziger theory," *Phys. Rev. D*, vol. 84, p. 065039, Sep 2011. 50

[74] L. C. Ferreira, A. D. Pereira, and R. F. Sobreiro, "Hints of confinement and deconfinement in yang-mills-chern-simons theories in the maximal abelian gauge," *Physical Review D*, vol. 101, no. 10, p. 105022, 2020. 50, 62, 63, 132

[75] D. Dudal, J. Gracey, V. Lemes, M. Sarandy, R. Sobreiro, S. Sorella, and H. Verschelde, "Analytic study of the off-diagonal mass generation for yang-mills theories in the maximal abelian gauge," *Physical Review D*, vol. 70, no. 11, p. 114038, 2004. 50

[76] M. Capri, D. Dudal, M. Guimaraes, I. Justo, S. Sorella, and D. Vercauteren, "Su (2)× u (1) yang–mills theories in 3 d with higgs field and gribov ambiguity," *The European Physical Journal C*, vol. 73, pp. 1–14, 2013. 52, 58, 73

[77] M. Capri, D. Dudal, M. Guimaraes, I. Justo, S. Sorella, and D. Vercauteren, "The (ir-) relevance of the gribov ambiguity in su (2)× u (1) gauge theories with fundamental higgs matter," *Annals of Physics*, vol. 343, pp. 72–86, 2014. 52, 58, 73

[78] I. Justo, M. Capri, D. Dudal, A. Gómez, M. Guimaraes, S. Sorella, and D. Vercauteren, "Confinement interpretation in a yang-mills+ higgs theory when considering gribov's ambiguity," *arXiv preprint arXiv:1506.09021*, 2015. 52, 58

[79] S. Deser, R. Jackiw, and S. Templeton, "Three-dimensional massive gauge theories," *Physical Review Letters*, vol. 48, no. 15, p. 975, 1982. 53, 63

[80] S. Deser, R. Jackiw, and S. Templeton, "Topologically massive gauge theories," *Annals of Physics*, vol. 281, no. 1-2, pp. 409–449, 2000. 53, 63

[81] F. Canfora, P. Pais, and P. Salgado-Rebolledo, "Gribov gap equation at finite temperature," *The European Physical Journal C*, vol. 74, no. 5, p. 2855, 2014. 54, 55, 60, 61, 76, 89, 96, 99, 132, 133

[82] C. P. Felix and C. W. Kao, "Implications of the topological chern-simons mass in the gap equation," *Physical Review D*, vol. 105, no. 9, p. 094016, 2022. 56

[83] R. K. Ellis, W. J. Stirling, and B. R. Webber, *QCD and collider physics*. Cambridge university press, 2003. 58, 60

[84] M. Capri, D. Dudal, A. Gomez, M. Guimaraes, I. Justo, and S. Sorella, "A study of the higgs and confining phases in euclidean su (2) yang–mills theories in 3 d by taking into account the gribov horizon," *The European Physical Journal C*, vol. 73, pp. 1–8, 2013. 61, 73, 132

[85] M. Capri, D. Dudal, A. Gomez, M. Guimaraes, I. Justo, S. Sorella, and D. Vercauteren, "Semiclassical analysis of the phases of 4 d s u (2) higgs gauge systems with cutoff at the gribov horizon," *Physical Review D*, vol. 88, no. 8, p. 085022, 2013. 61, 73, 132

[86] M. Capri, V. Lemes, R. Sobreiro, S. Sorella, and R. Thibes, "Influence of the gribov copies on the gluon and ghost propagators in euclidean yang-mills theory in the maximal abelian gauge," *Physical Review D*, vol. 72, no. 8, p. 085021, 2005. 63

[87] L. C. Ferreira, "Gribov ambiguities in yang-mills-chern-simons theories in the maximal abelian gauge, masters degree dissertation," 2019. 63

[88] A. D. Pereira and R. F. Sobreiro, "On the elimination of infinitesimal gribov ambiguities in non-abelian gauge theories," *The European Physical Journal C*, vol. 73, pp. 1–18, 2013. 67

[89] A. D. Pereira and R. F. Sobreiro, "Gribov ambiguities at the landau-maximal abelian interpolating gauge," *The European Physical Journal C*, vol. 74, pp. 1–15, 2014. 67

[90] P. M. Lavrov and O. Lechtenfeld, "Gribov horizon beyond the landau gauge," *Physics Letters B*, vol. 725, no. 4-5, pp. 386–388, 2013. 67

[91] P. Y. Moshin and A. A. Reshetnyak, "Field-dependent brst–antibrst transformations in yang–mills and gribov–zwanziger theories," *Nuclear Physics B*, vol. 888, pp. 92–128, 2014. 67

[92] D. Dudal, S. Sorella, N. Vandersickel, and H. Verschelde, "Gribov no-pole condition, zwanziger horizon function, kugo-ojima confinement criterion, boundary conditions, brst breaking and all that," *Physical Review D*, vol. 79, no. 12, p. 121701, 2009. 67

[93] A. Cucchieri, D. Dudal, T. Mendes, and N. Vandersickel, "Brst-symmetry breaking and bose-ghost propagator in lattice minimal landau gauge," *Physical Review D*, vol. 90, no. 5, p. 051501, 2014. 67

[94] M. Capri, D. Fiorentini, and S. Sorella, "Gribov horizon and non-perturbative brst symmetry in the maximal abelian gauge," *Physics Letters B*, vol. 751, pp. 262–271, 2015. 67

[95] A. J. Gomez, S. Gonzalez, and S. P. Sorella, "Yang–mills–chern–simons system in the presence of a gribov horizon with fundamental higgs matter," *Journal of Physics A: Mathematical and Theoretical*, vol. 49, no. 6, p. 065401, 2016. 73

[96] A. M. Polyakov, "Quark confinement and topology of gauge theories," *Nuclear Physics B*, vol. 120, no. 3, pp. 429–458, 1977. 74

[97] D. Dudal, J. Gracey, S. Sorella, N. Vandersickel, and H. Verschelde, "Landau gauge gluon and ghost propagators in the refined gribov-zwanziger framework in 3 dimensions," *Physical Review D*, vol. 78, no. 12, p. 125012, 2008. 75

[98] M. Le Bellac, *Thermal field theory*. Cambridge university press, 2000. 76

[99] J. I. Kapusta and C. Gale, *Finite-temperature field theory: Principles and applications*. Cambridge university press, 2006. 76

[100] R. Alkofer and L. Von Smekal, "The infrared behaviour of qcd green's functions: confinement, dynamical symmetry breaking, and hadrons as relativistic bound states," *Physics Reports*, vol. 353, no. 5-6, pp. 281–465, 2001. 87

[101] K. Amemiya and H. Suganuma, "Off-diagonal gluon mass generation and infrared abelian dominance in the maximally abelian gauge in lattice qcd," *Physical Review D*, vol. 60, no. 11, p. 114509, 1999. 110

[102] A. D. P. G. P. D. M. van Egmond, L. C. Ferreira and S. P. Sorella, "Euclidean yang-mills massive model in the maximal abelian gauge," *arxiv*, 2023. 113, 117

[103] V. Shtabovenko, R. Mertig, and F. Orellana, "New developments in feyncalc 9.0," *Computer Physics Communications*, vol. 207, pp. 432–444, 2016. 116

[104] J. Liao and E. Shuryak, "Strongly coupled plasma with electric and magnetic charges," *Physical Review C*, vol. 75, no. 5, p. 054907, 2007. 133

[105] R. D. Pisarski, "Towards a theory of the semi-quark gluon plasma," *Nuclear Physics B-Proceedings Supplements*, vol. 195, pp. 157–198, 2009. 133

[106] Y. Hidaka and R. D. Pisarski, "Suppression of the shear viscosity in a "semi"-quark-gluon plasma," *Physical Review D*, vol. 78, no. 7, p. 071501, 2008. 133

[107] R. D. Pisarski, K. Kashiwa, and V. Skokov, "Quasi-particle and matrix models of the semi quark gluon plasma," *Nuclear Physics A*, vol. 904, pp. 973c–976c, 2013. 133

[108] M. Capri, M. Guimaraes, I. Justo, L. Palhares, and S. Sorella, "Properties of the faddeev-popov operator in the landau gauge, matter confinement, and soft brst breaking," *Physical Review D*, vol. 90, no. 8, p. 085010, 2014. 135

[109] M. Capri, S. Sorella, and R. Terin, "All order renormalizable refined gribov-

zwanziger model with brst invariant fermionic horizon function in linear covariant gauges," *Physical Review D*, vol. 104, no. 5, p. 054048, 2021. 135